Physiological Age, Health costs and their interrelation

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Outline

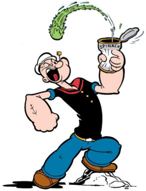
Plan of the presentation:

1. Motivation and main objectives
2. Phase-type lifetime and aging
3. Health states and cost information
4. Implications for underwriting
5. Conclusion and perspectives

Motivation and main objectives

Heterogeneity in a population:

same age different health



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Financial implications for life, pension and health insurance

Motivation and main objectives

Quantification of health

- Frailty models: health is s.t. unobservable mortality risk factor, which is given at birth and does not change with age
- Underwriting criteria: observable facts
- Phase-type aging model by Lin&Liu'07: Markovian model with unobservable health states
- Meyricke&Sherris'13: for proper underwriting it is important to take into account observable and unobservable factors

Our objective

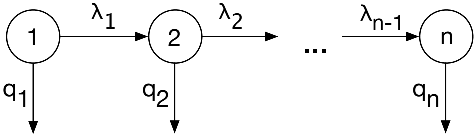
use observable information to characterise health states in the phase-type aging model

Phase-type lifetime and aging

$$\boxed{\textit{PH distributions are dense} \Rightarrow \textit{Lifetime} \sim PH(\underline{\alpha}, \mathbf{\Lambda})}$$

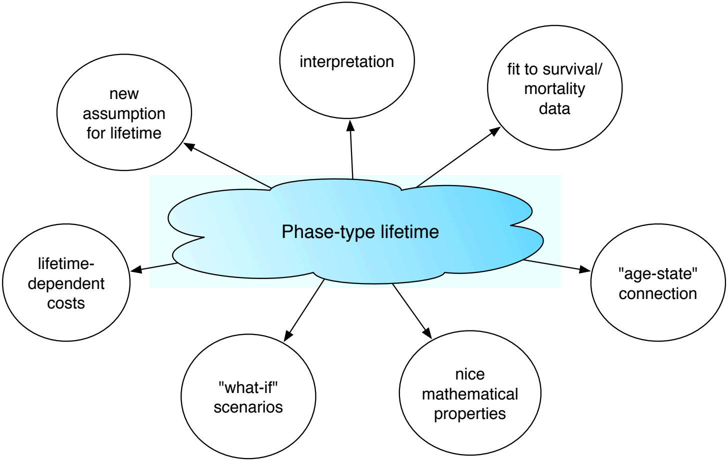
Phase-type aging model [Lin, Liu'07]

$\mathbf{\Lambda}$:

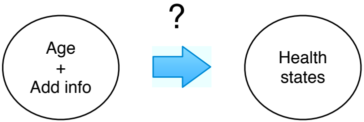


- Markov chain for human mortality, phases: *health states*
- Fit to mortality data, $\underline{\alpha} = [1\,0\ldots\,0]$
- Health state distribution at age x: $\underline{f}_x = \underline{\alpha} e^{\mathbf{\Lambda} x} / \underline{\alpha} e^{\mathbf{\Lambda} x} \mathbf{1}$
- Survival function for an individual aged x: $S_x(t) = \underline{f}_x e^{\mathbf{\Lambda} t} \mathbf{1}$

Advantages of the assumption



Health states and cost information



No additional information:

$$\text{Age } x \;\Rightarrow\; \text{health state distribution is } \underline{f}_x$$

Direct information: $\bar{e}$ – predicted life expectancy.

$$\bar{e} \;\Rightarrow\; \text{most likely health state } \; i^* : \; e^{(i^*+1)} \le \bar{e} \le e^{(i^*)},$$

$$e^{(i)} = \underline{\alpha}^{(i)} \sum_{k=0}^{\infty} k e^{\mathbf{\Lambda} k}(I - e^{\mathbf{\Lambda}})\mathbf{1}, \quad \underline{\alpha}^{(i)} : \; \alpha_j^{(j)} = 1, \; \alpha_i^{(j)} = 0, \; i \neq j.$$

$e^{(i)}$ is the expected life length in state i.

Health states and cost information

Indirect information: C_x – annual health cost at age x, Y_x – health state at age x. We are interested in

$$f_x(i|c) = \mathrm{P}[\, Y_x = i \mid C_x = c \,], \quad i = 1, \ldots, n.$$

We assume:

$$\mathrm{P}[\, C_x = c \mid Y_x = i \,] \approx \mathrm{P}[\, C = c \mid Y = i \,] = f(c|i)$$

This results in

$$f_x(i|c) \approx \frac{f(c|i) f_x(i)}{\sum_{j=1}^{n} f(c|j) f_x(j)}, \quad f_x(i) = \mathrm{P}[Y_x = i]$$

$\Rightarrow$ need to determine $f(c|i)$, $i = 1, \ldots, n$.

Health states and cost information

Properties of health cost data:

- Persistent with age
- Probability mass at zero

Denote $f(0|i) = \mathrm{P}[C = 0|Y = i],\ f(c|i, c > 0) = \mathrm{P}[C = c|Y = i, C > 0]$

$$f(c|i) = \begin{cases} f(0|i), & c = 0 \\ (1 - f(0|i))f(c|i, c > 0), & c > 0 \end{cases}$$

Least squares to determine $f(0|i)$ **and** $f(c|i, c > 0)$:

$$\min_{\underline{\phi}} \sum_x n_x(\underline{f}_x \underline{\phi} - \hat{\psi}_x)^2,\ \underline{\phi} = [\phi_i]_{i=1,\ldots,n} \quad + \quad \text{constraints}$$

ϕ_i – unknown state dependent function, $\hat{\psi}_x$ – observed age dependent function, n_x – weight of population of age x in the observed population

Health states and cost information

Lognormal costs. Numerical illustration.

$$\mathrm{P}[\, C = c \mid Y = i, C > 0\,] \sim log\mathcal{N}(\mu_i, \sigma_i^2)$$

Step I. Least squares to determine the shapes

- $f(c|i, c > 0) : \phi_i = F(c|i, c > 0),\ \hat{\psi}_x = [\hat{F}_x(c|c > 0)]_x$ with

$$0 \leq F(c|i, c > 0) \leq F(c + 1|i, c > 0) \leq 1$$

- $f(0|i)$: $\phi_i = f(0|i)_i,\ \hat{\psi}_x = \mathrm{P}[\, C_x = 0\,]$ with

$$0 \leq f(0|i + 1) \leq f(0|i) \leq 1$$

C_x – annual health care cost at age x

- μ_i and σ_i^2: similar approach for $\log C_x$ + technical constraints

Health states and cost information

Step II. Least squares for parametric functions

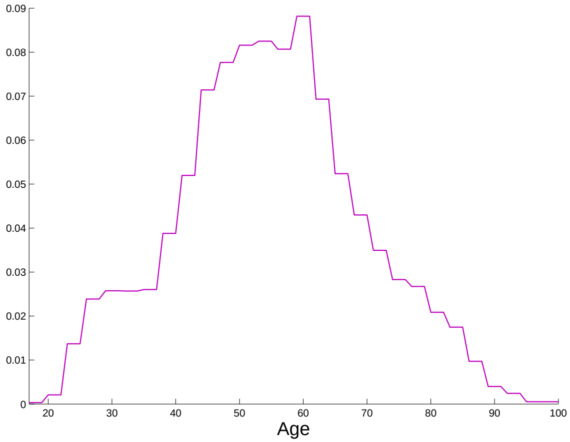
$$\begin{array}{ll} f(0|i): & \rho_1(i|a_0, a_1, a_2) = \exp(-a_0 i - a_1) + a_2 \\ \mu_i: & \rho_2(i|b_0, b_1) = b_0 i + b_1 \\ \sigma_i^2: & \rho_3(i|d) = d \end{array},$$

Data.

- Phase-type aging model: 1911 Swedish cohort of males
- Health care costs database 2008: 15334 individual records for males
- Age: 17-100
- Maximal observed cost: 200 000$
- Age interval: 3 years, cost interval: 200$

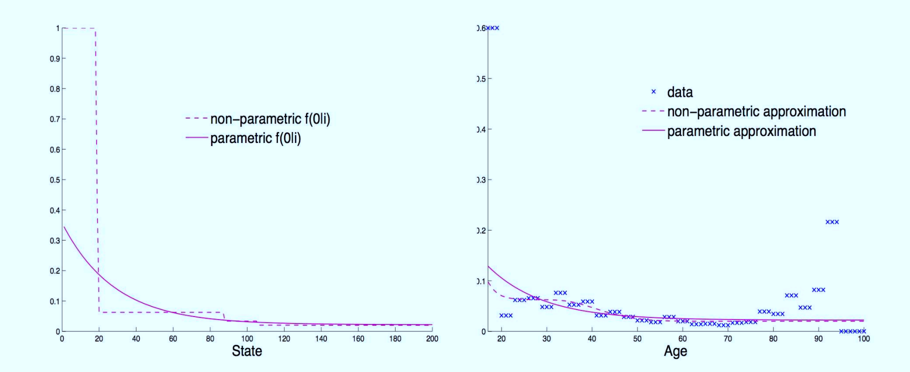
Health states and cost information

Number of people in each age group, n_x



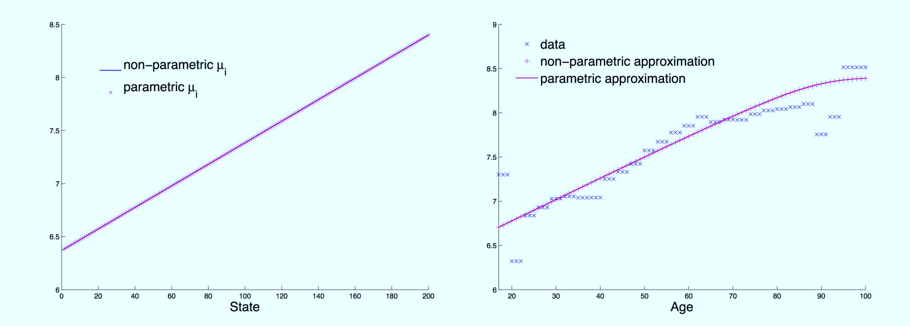
Health states and cost information

Probability mass at zero

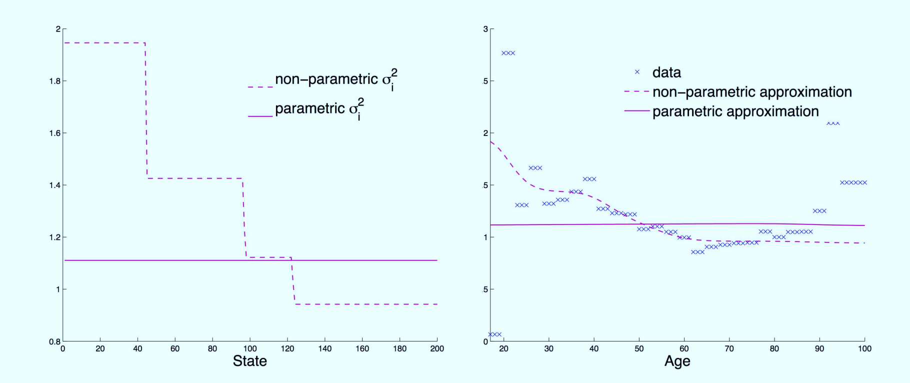


Health states and cost information

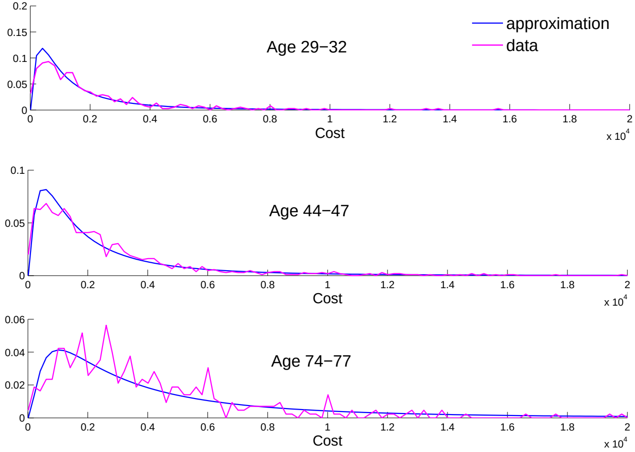
Expectation of lognormal cost



Variance of lognormal cost



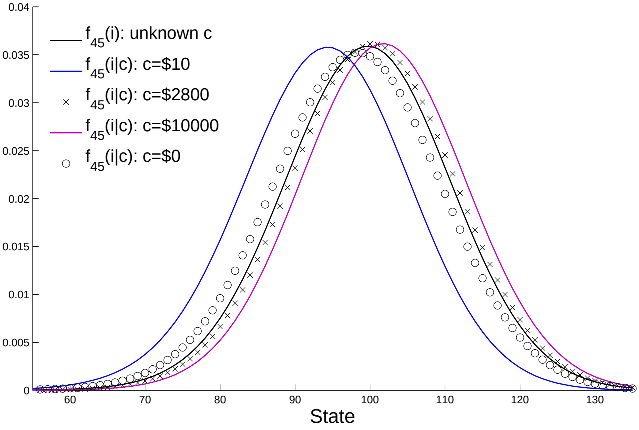
Distribution of costs: fitting outcome



Health states and cost information

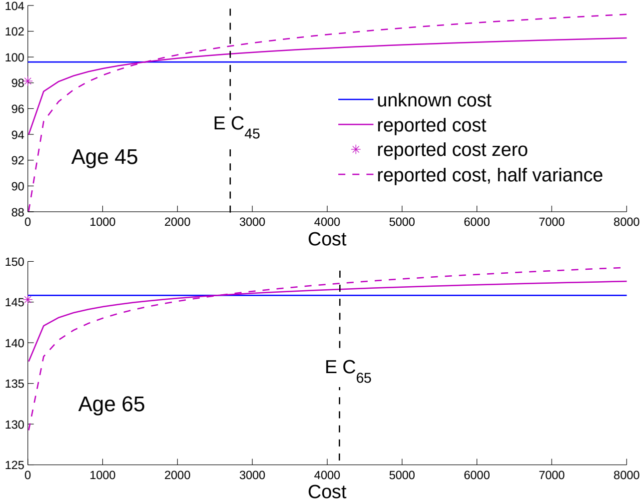
Distribution of health state

$$f_x(i|c) = \mathrm{P}[\, Y_x = i \mid C_x = c\,], \quad f_x(i) = \mathrm{P}[\, Y_x = i\,], \quad i = 1, \ldots, n$$



Health states and cost information

Expected health state



Implications for underwriting

Net Present Value of a health care contract

$$S = \sum_{i=1}^{L} v^{i-1} X_i$$

- L – individual's remaining lifetime, $L \sim PH(\underline{f}_x, \mathbf{\Lambda})$
- v – positive constant for discounting
- X_i – health care cost in year i, *Markov Reward Model* $(\{A\}\&D, \underline{\mathcal{W}}, P)$

  A = health states, $\{D\}$ = absorbing state
  If $j \in A$, then $X_i \sim \mathcal{W}_j$, If $j \in \{D\}$, then $X_i = 0$
  State transition matrix

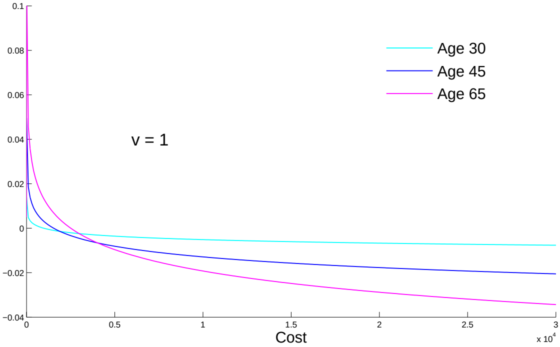
$$P = \begin{bmatrix} e^{\mathbf{\Lambda}} & \underline{y} \\ \mathbf{0} & 1 \end{bmatrix}, \quad \underline{y} = \mathbf{1} - e^{\mathbf{\Lambda}}\mathbf{1}$$

Implications for underwriting

Expected Net Present Value

$$E[S] = \underline{f}_x (I - ve^{\mathbf{\Lambda}})^{-1} E[\underline{\mathcal{W}}]$$

Change of $E[S]$ given cost information, %



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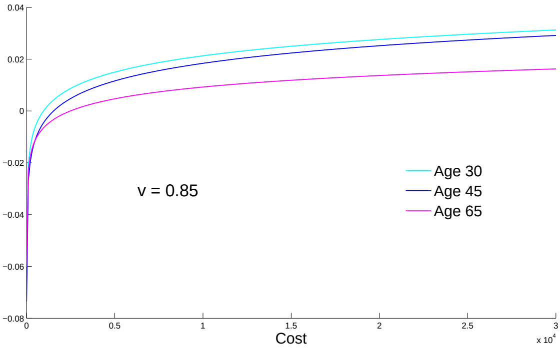

Conclusion

Main results

- extension of PH-aging model: method to capture the effect of heterogeneity based on actual health care costs in the *past year*
- impact on the NPV of lifelong health care contracts

Perspectives

- further extension of PH-aging model: method to capture the effect of heterogeneity based on *history* of actual health care costs
- include other types of information

Conclusion

Thank you for your attention!