Physiological Age, Health costs and their interrelation

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Plan of the presentation:

1. Motivation and main objectives
2. Phase-type lifetime and aging
3. Health states and cost information
4. Implications for underwriting
5. Conclusion and perspectives
Motivation and main objectives

Heterogeneity in a population:

same age different health

Financial implications for life, pension and health insurance
Motivation and main objectives

Quantification of health

- Frailty models: health is s.t. unobservable mortality risk factor, which is given at birth and does not change with age
- Underwriting criteria: observable facts
- Phase-type aging model by Lin&Liu’07: Markovian model with unobservable health states
- Meyricke&Sherris’13: for proper underwriting it is important to take into account observable and unobservable factors

Our objective

use observable information to characterise health states in the phase-type aging model
PH distributions are dense \( \Rightarrow \) Lifetime \( \sim \) PH(\( \alpha \), \( \Lambda \))

**Phase-type aging model** [Lin, Liu’07]

- Markov chain for human mortality, phases: *health states*
- Fit to mortality data, \( \alpha = [1 \ 0 \ldots \ 0] \)
- Health state distribution at age \( x \): \( f_x = \frac{\alpha e^{\Lambda x}}{\alpha e^{\Lambda x}1} \)
- Survival function for an individual aged \( x \): \( S_x(t) = f_x e^{\Lambda t}1 \)
Advantages of the assumption

- New assumption for lifetime interpretation
- Fit to survival/mortality data
- "What-if" scenarios
- Nice mathematical properties
- "Age-state" connection
- Lifetime-dependent costs

Phase-type lifetime
Health states and cost information

No additional information:

\[
\text{Age } x \Rightarrow \text{health state distribution is } f_x
\]

Direct information: \(\bar{e}\) – predicted life expectancy.

\[
\bar{e} \Rightarrow \text{most likely health state } i^* : e^{(i^*+1)} \leq \bar{e} \leq e^{(i^*)},
\]

\[
e^{(i)} = \alpha^{(i)} \sum_{k=0}^{\infty} ke^{\Lambda k} (I - e^{\Lambda})1, \quad \alpha^{(i)} : \alpha^{(j)} = 1, \alpha^{(i)} = 0, \quad i \neq j.
\]

\(e^{(i)}\) is the expected life length in state \(i\).
Health states and cost information

**Indirect information:** $C_x$ – annual health cost at age $x$, $Y_x$ – health state at age $x$. We are interested in

$$f_x(i|c) = P[Y_x = i \mid C_x = c], \quad i = 1, \ldots, n.$$  

We assume:

$$P[C_x = c \mid Y_x = i] \approx P[C = c \mid Y = i] = f(c|i)$$

This results in

$$f_x(i|c) \approx \frac{f(c|i)f_x(i)}{n \sum_{j=1}^{n} f(c|j)f_x(j)}, \quad f_x(i) = P[Y_x = i]$$

$\Rightarrow$ need to determine $f(c|i), \ i = 1, \ldots, n.$
Properties of health cost data:

- Persistent with age
- Probability mass at zero

Denote \( f(0|i) = P[C = 0| Y = i] \), \( f(c|i, c > 0) = P[C = c| Y = i, C > 0] \)

\[
f(c|i) = \begin{cases} 
  f(0|i), & c = 0 \\
  (1 - f(0|i))f(c|i, c > 0), & c > 0
\end{cases}
\]

Least squares to determine \( f(0|i) \) and \( f(c|i, c > 0) \):

\[
\min_{\phi} \sum_x n_x (f_x \phi - \hat{\psi}_x)^2, \phi = [\phi_i]_{i=1,...,n} + \text{ constraints}
\]

\( \phi_i \) – unknown state dependent function, \( \hat{\psi}_x \) – observed age dependent function, \( n_x \) – weight of population of age \( x \) in the observed population
Lognormal costs. Numerical illustration.

\[
P[ C = c \mid Y = i, C > 0] \sim \log N(\mu_i, \sigma^2_i)
\]

**Step 1.** Least squares to determine the shapes

\[ f(c\mid i, c > 0) : \phi_i = F(c\mid i, c > 0), \quad \hat{\psi}_x = [\hat{F}_x(c\mid c > 0)]_x \quad \text{with} \]

\[ 0 \leq F(c\mid i, c > 0) \leq F(c + 1\mid i, c > 0) \leq 1 \]

\[ f(0\mid i) : \phi_i = f(0\mid i), \quad \hat{\psi}_x = P[ C_x = 0 ] \quad \text{with} \]

\[ 0 \leq f(0\mid i + 1) \leq f(0\mid i) \leq 1 \]

\( C_x \) – annual health care cost at age \( x \)

\( \mu_i \) and \( \sigma^2_i \): similar approach for \( \log C_x \) + technical constraints
Health states and cost information

**Step II.** Least squares for parametric functions

\[ f(0| i) : \quad \rho_1(i| a_0, a_1, a_2) = \exp(-a_0 i - a_1) + a_2 \]
\[ \mu_i : \quad \rho_2(i| b_0, b_1) = b_0 i + b_1 \]
\[ \sigma_i^2 : \quad \rho_3(i| d) = d \]

**Data.**

- Phase-type aging model: 1911 Swedish cohort of males
- Health care costs database 2008: 15334 individual records for males
- Age: 17-100
- Maximal observed cost: 200 000$
- Age interval: 3 years, cost interval: 200$
Number of people in each age group, $n_x$
Health states and cost information

Probability mass at zero

![Graph showing probability mass at zero vs state and age](image)
Expectation of lognormal cost

\[ \text{non-parametric } \mu_i \]
\[ \text{parametric } \mu_i \]
Health states and cost information

Variance of lognormal cost

![Graph](Image)
Health states and cost information

Distribution of costs: fitting outcome

Approximation vs. data for different age groups:
- Age 29–32
- Age 44–47
- Age 74–77

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Health states and cost information

Distribution of health state

\[ f_x(i|c) = P[Y_x = i | C_x = c], \quad f_x(i) = P[Y_x = i], \quad i = 1, \ldots, n \]
Expected health state

Age 45

Age 65

Cost

unknown cost
reported cost
reported cost zero
reported cost, half variance
Implications for underwriting

Net Present Value of a health care contract

\[ S = \sum_{i=1}^{L} v^{i-1} X_i \]

- \( L \) – individual’s remaining lifetime, \( L \sim PH(f_x, \Lambda) \)
- \( v \) – positive constant for discounting
- \( X_i \) – health care cost in year \( i \), \( Markov \ Reward \ Model(\{A\&D, W, P) \)

\( A = \text{health states}, \{D\} = \text{absorbing state} \)
If \( j \in A \), then \( X_i \sim W_j \), If \( j \in \{D\} \), then \( X_i = 0 \)
State transition matrix

\[ P = \begin{bmatrix} e^\Lambda & y \\ 0 & 1 \end{bmatrix}, \ y = 1 - e^\Lambda 1 \]
Implications for underwriting

Expected Net Present Value

\[ E[S] = f_x(l - ve^\Lambda)^{-1} E[W] \]

Change of \( E[S] \) given cost information, %
Implications for underwriting

Expected Net Present Value

\[ E[S] = f_x(I - ve^\Lambda)^{-1} E[\mathcal{W}] \]

Change of \( E[S] \) given cost information, %
Conclusion

Main results

- extension of PH-aging model: method to capture the effect of heterogeneity based on actual health care costs in the *past year*
- impact on the NPV of lifelong health care contracts

Perspectives

- further extension of PH-aging model: method to capture the effect of heterogeneity based on *history* of actual health care costs
- include other types of information
Thank you for your attention!