Actuarial Model Outcome Optimal Fit
AMOOF 3.0

Presented to ARC 2014

Presenters: Jim Smigaj Dr. Paul H Johnson, Jr.

Dr. John Anvik

Dr. Yvonne Chueh

Introduction

- Modeling single and mixed Probability Density Functions (pdfs)
- Integrating tail VaR and TVaR of the found model curves
- Optimization of the log-likelihood function
- Interactive and Graphical user interface
- Correcting small-sample bias correction for the maximum likelihood estimations

Introduction ~cont'd

- **Version 3.0** adds cross-mixed pdfs (22 times 22 pdfs) to allow mixing different density families, increases the speed of improper integral calculus, adds a real-time check on numeric accuracy, and includes a public website for the user to download the program and report testing issues for future improvement.
- **Version 2.0** was sponsored by the Actuarial Foundation

Projects Using AMOOF 3

- **Stochastic Modeling Efficiency**
- Loss Models Excel Tools: Simulator, Fitter, and Tester Tools
- Real World Projects

Analyzing the Seahawks Offensive Play-Calling During The 2013 Regular Season

Adam Brand

May 2014

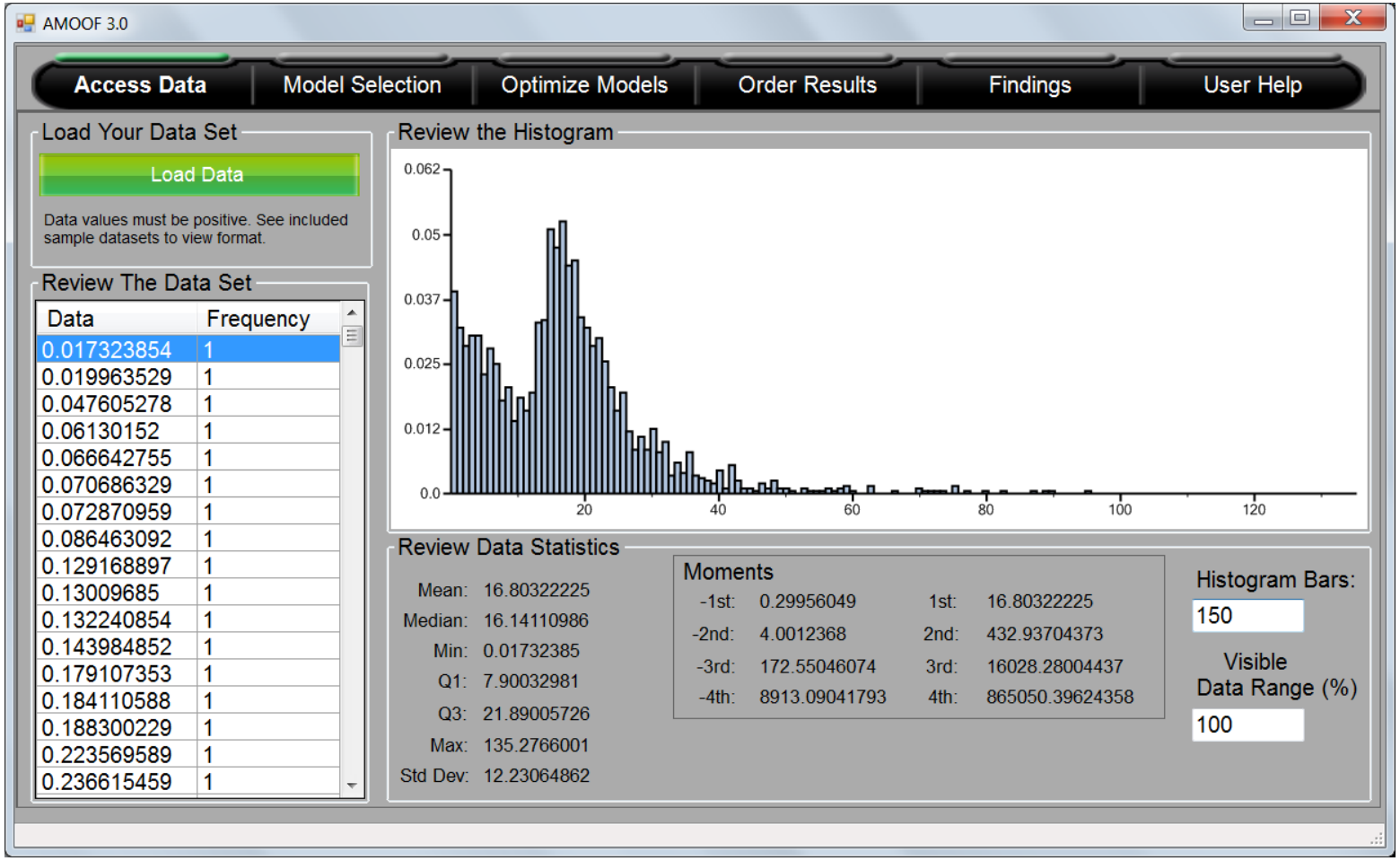
MODELLING LION'S ROCK

Ben Squire
Micah Darnall
Paul Carpenter
Christian Chmielewski

AMOOF 3.0 Demonstration

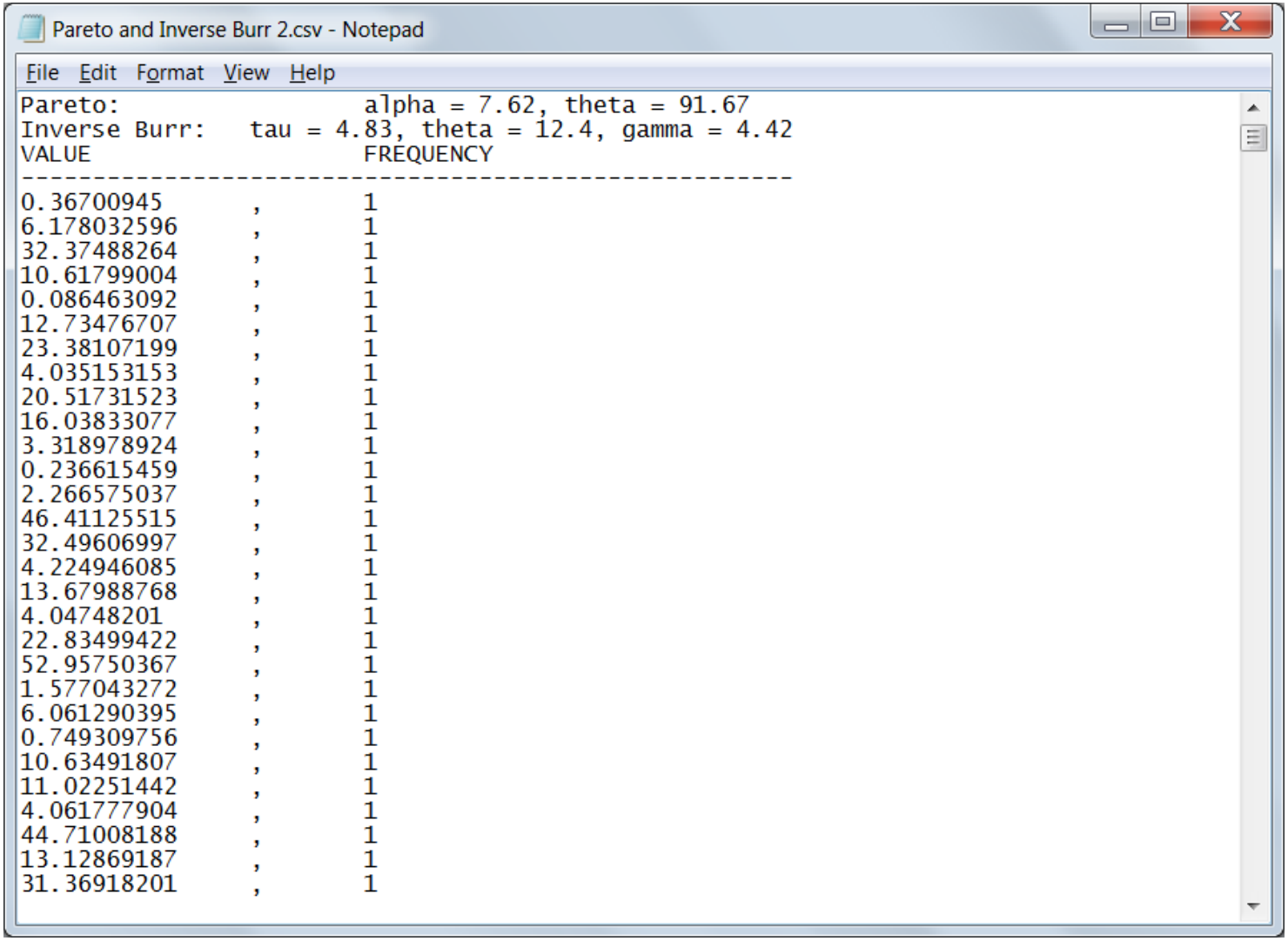
James Smigaj

Central Washington University



ACCESS DATA PANEL

Data Format


Pareto and Inverse Burr 2.csv - Notepad
File Edit Format View Help
Pareto: alpha = 7.62, theta = 91.67
Inverse Burr: tau = 4.83, theta = 12.4, gamma = 4.42
VALUE FREQUENCY
0.36700945 , 1
6.178032596 , 1
32.37488264 , 1
10.61799004 , 1
0.086463092 , 1
12.73476707 , 1
23.38107199 , 1
4.035153153 , 1
20.51731523 , 1
16.03833077 , 1
3.318978924 , 1
0.236615459 , 1
2.266575037 , 1
46.41125515 , 1
32.49606997 , 1
4.224946085 , 1
13.67988768 , 1
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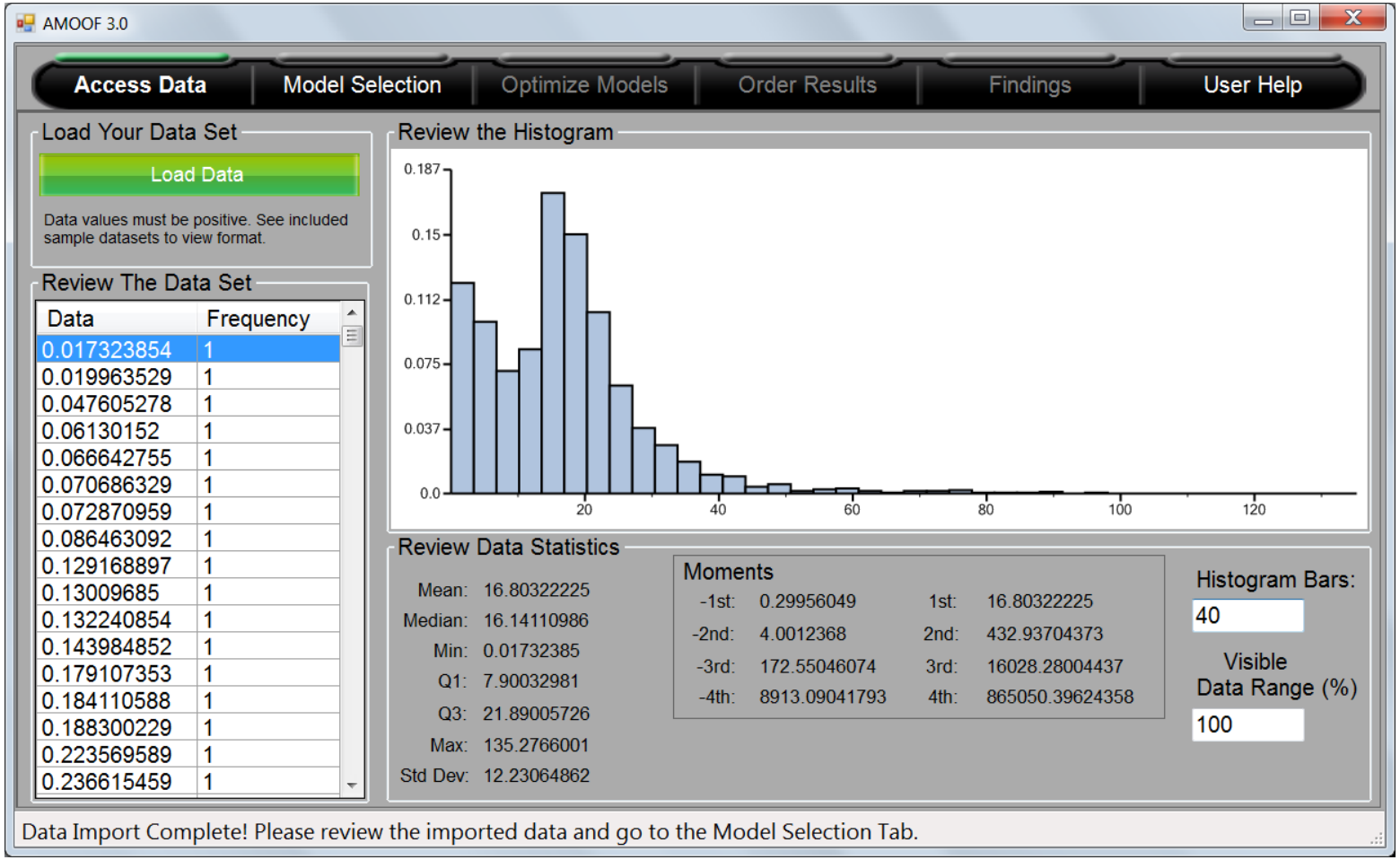
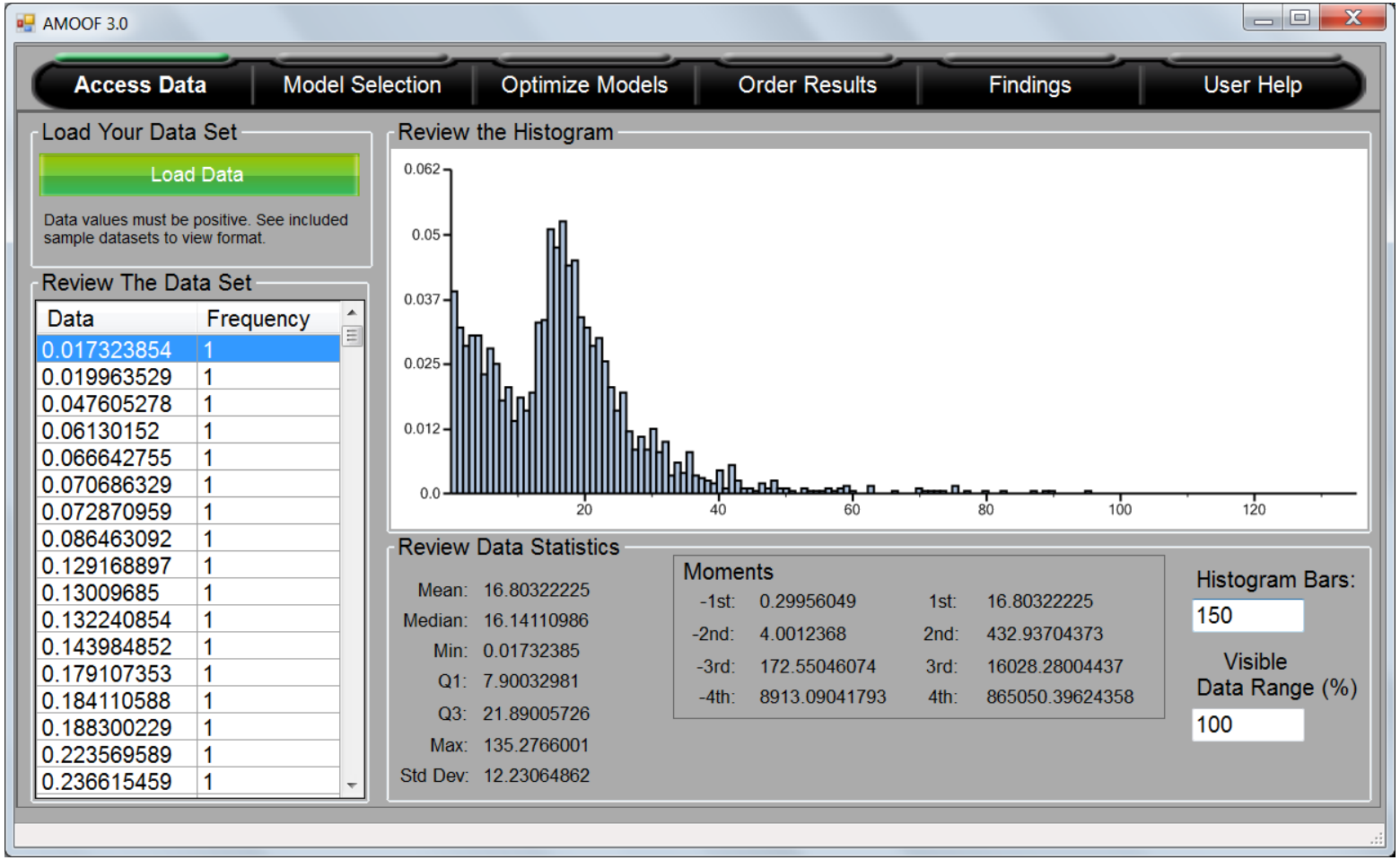
22.83499422 , 1
52.95750367 , 1
1.577043272 , 1
6.061290395 , 1
0.749309756 , 1
10.63491807 , 1
11.02251442 , 1
4.061777904 , 1
44.71008188 , 1
13.12869187 , 1
31.36918201 , 1

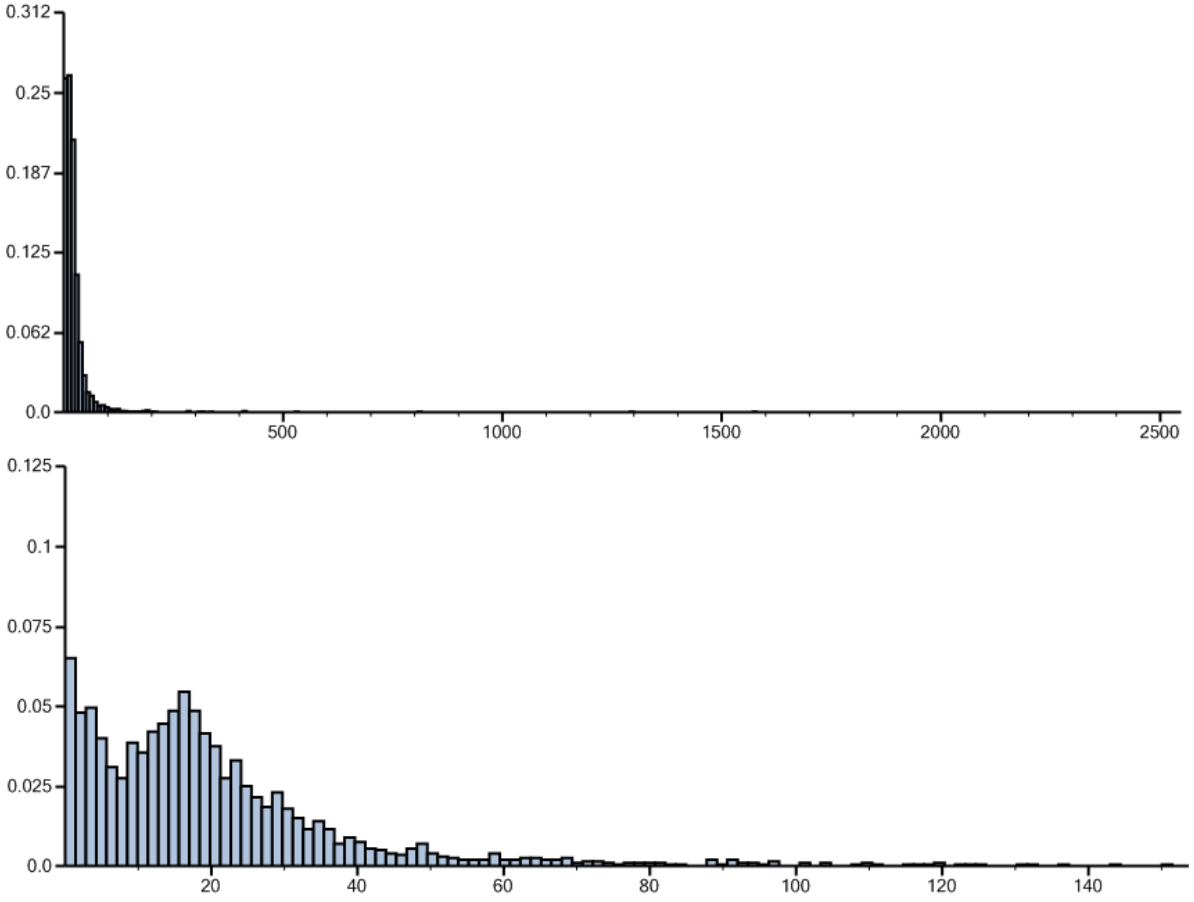
Load Dataset



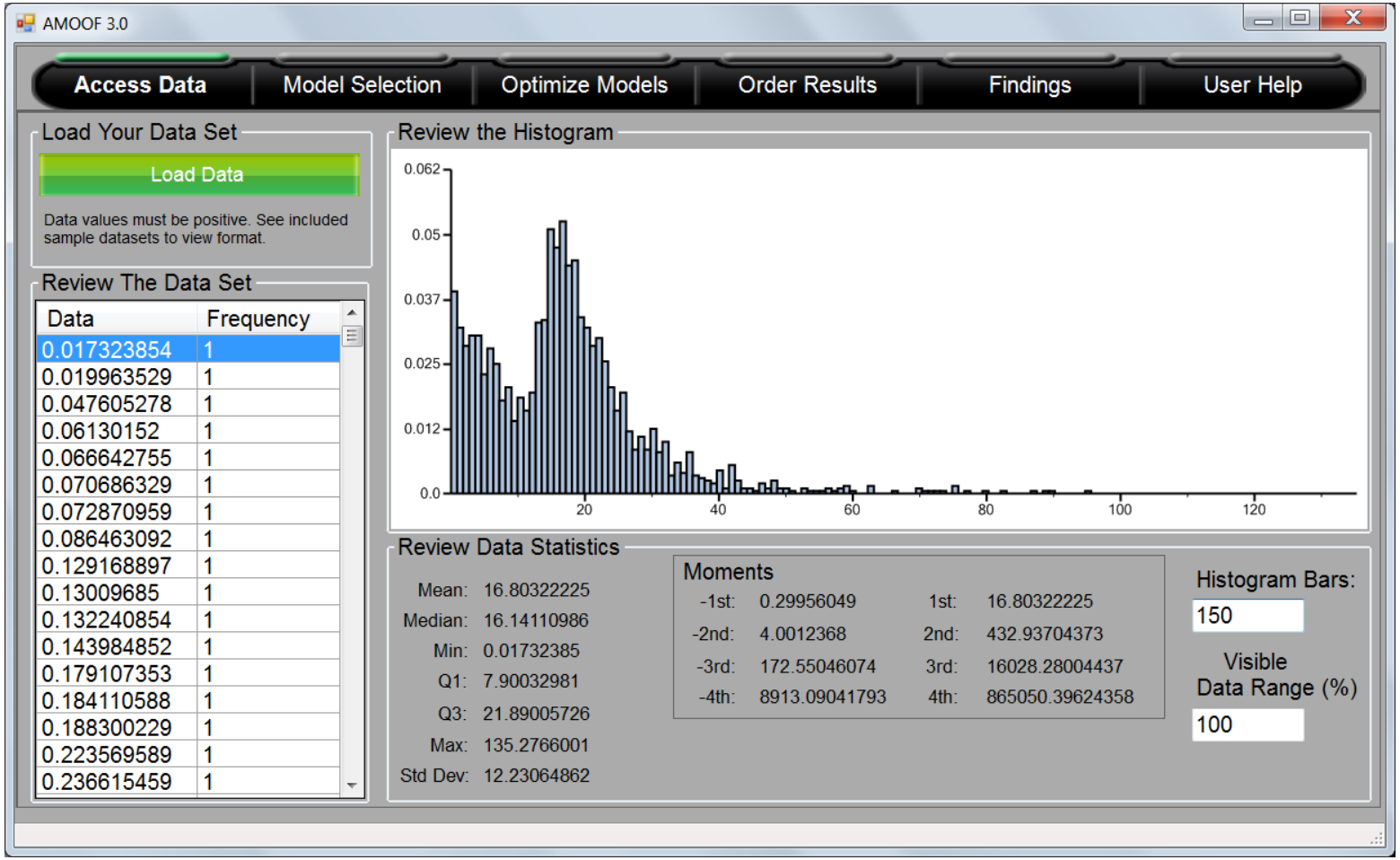
Histogram bars

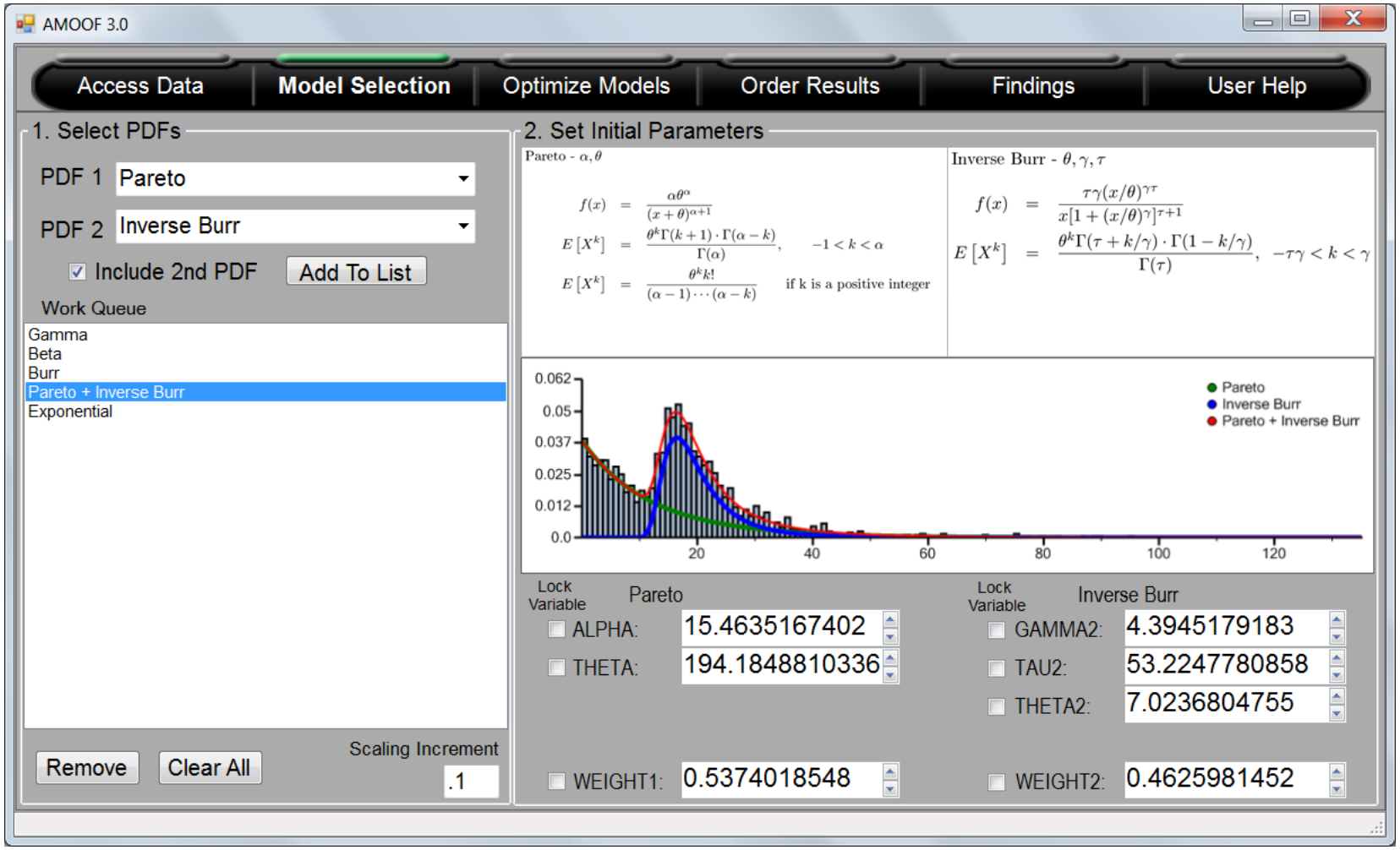


Histogram range



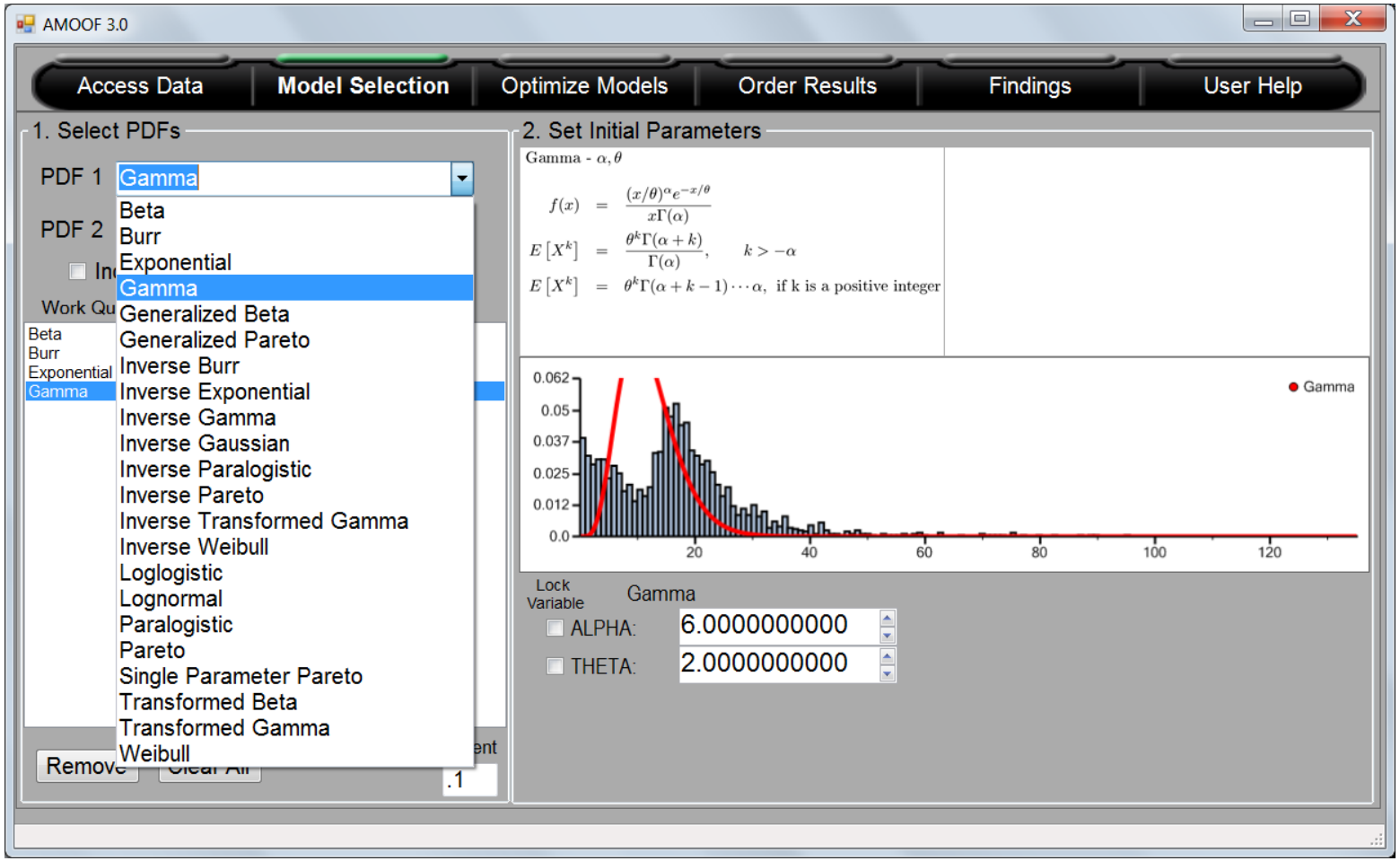
View dataset statistics



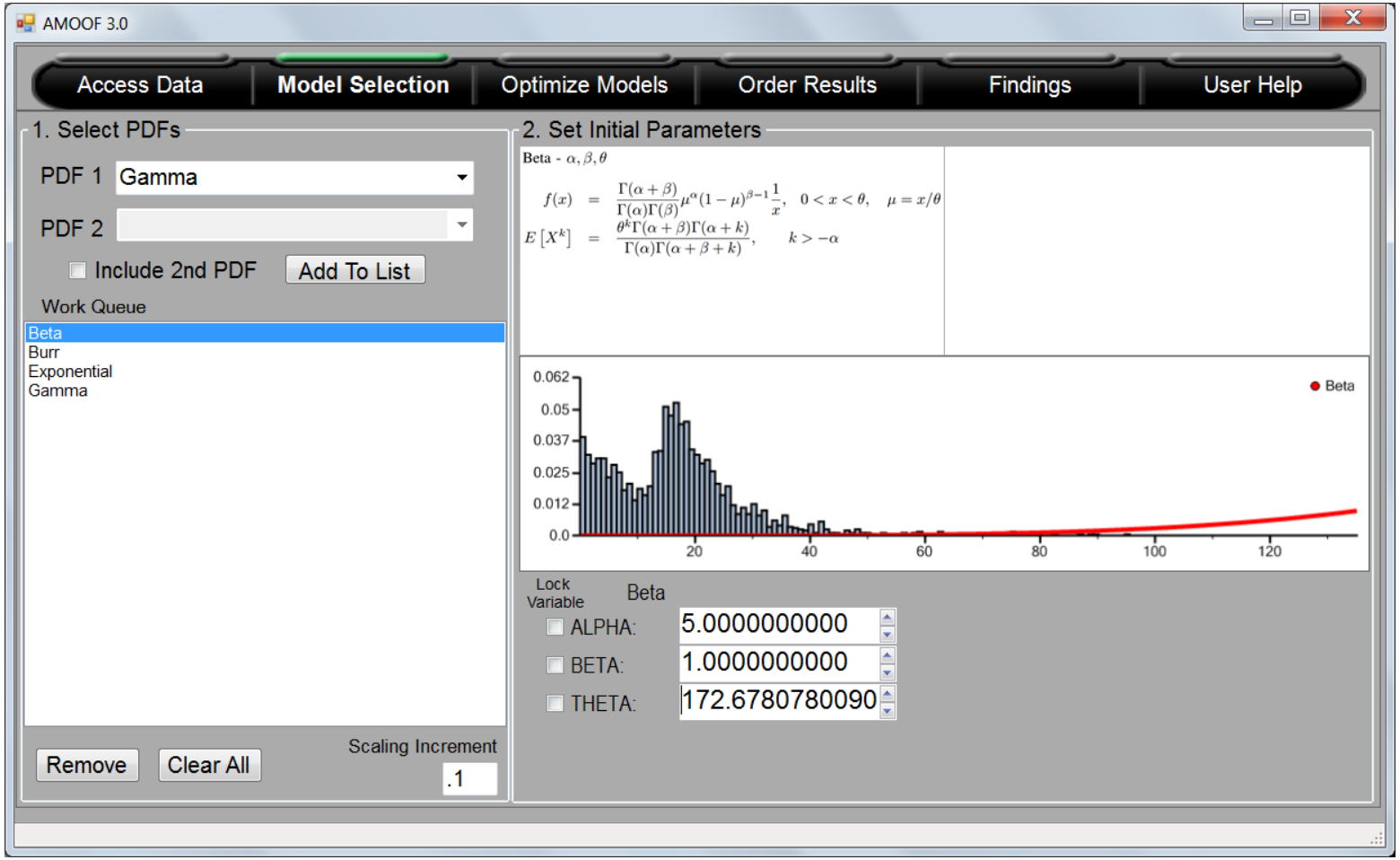


MODEL SELECTION PANEL

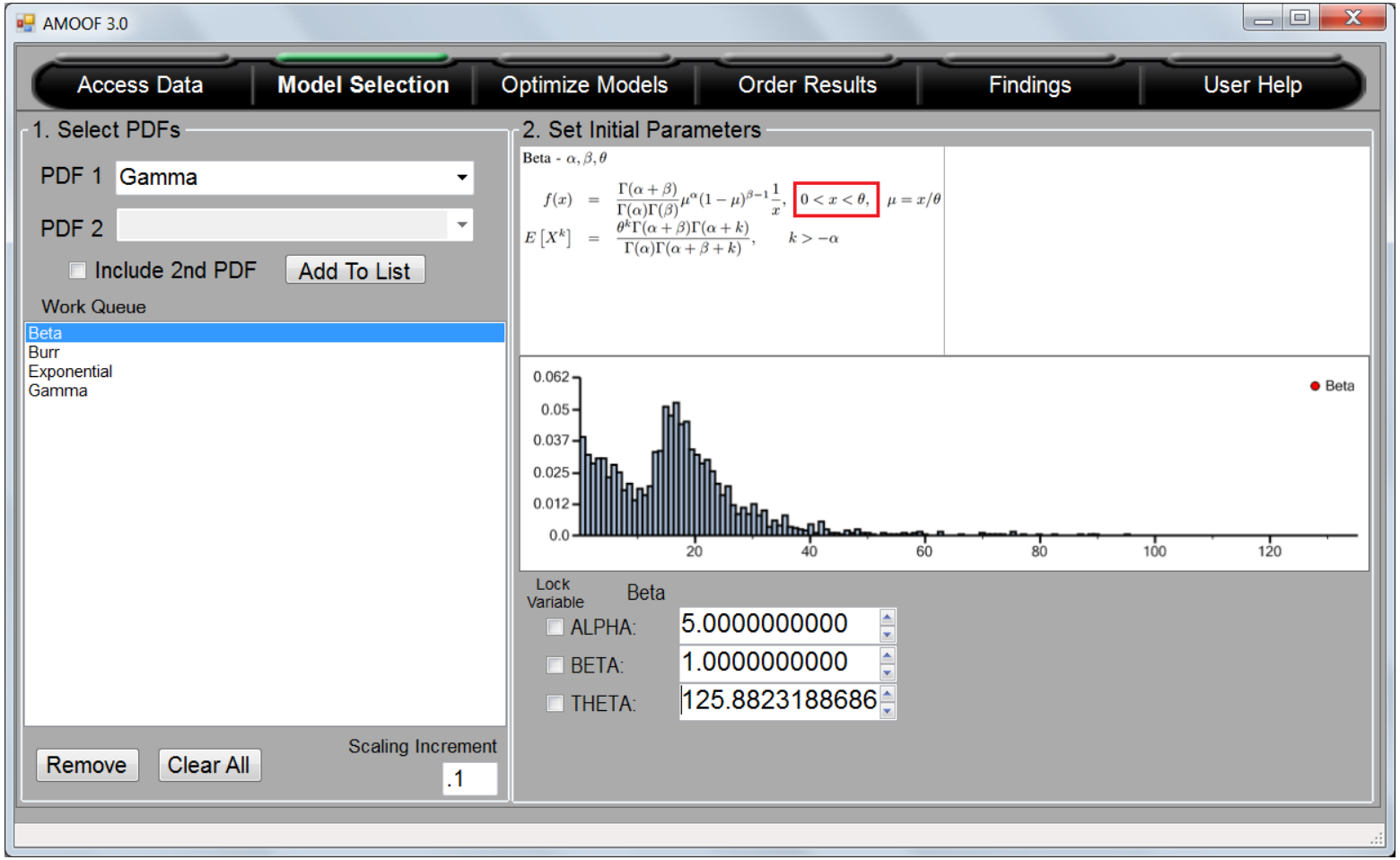
Select pdfs


AMOOF 3.0
Access Data
Model Selection
Optimize Models
Order Results
Findings
User Help
1. Select PDFs
PDF 1
Gamma
PDF 2
Beta
Burr
Exponential
Gamma
Generalized Beta
Generalized Pareto
Inverse Burr
Inverse Exponential
Inverse Gamma
Inverse Gaussian
Inverse Paralogistic
Inverse Pareto
Inverse Transformed Gamma
Inverse Weibull
Loglogistic
Lognormal
Paralogistic
Pareto
Single Parameter Pareto
Transformed Beta
Transformed Gamma
Weibull
Work Qu
Remove
.1
2. Set Initial Parameters
Gamma - α, θ
$f(x) = \frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)}$
$E[X^k] = \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(\alpha)}, \quad k > -\alpha$
$E[X^k] = \theta^k \Gamma(\alpha+k-1)\cdots\alpha$, if k is a positive integer
0.062
0.05
0.037
0.025
0.012
0.0
20
40
60
80
100
120
Gamma
Lock Variable
Gamma
ALPHA:
6.0000000000
THETA:
2.0000000000

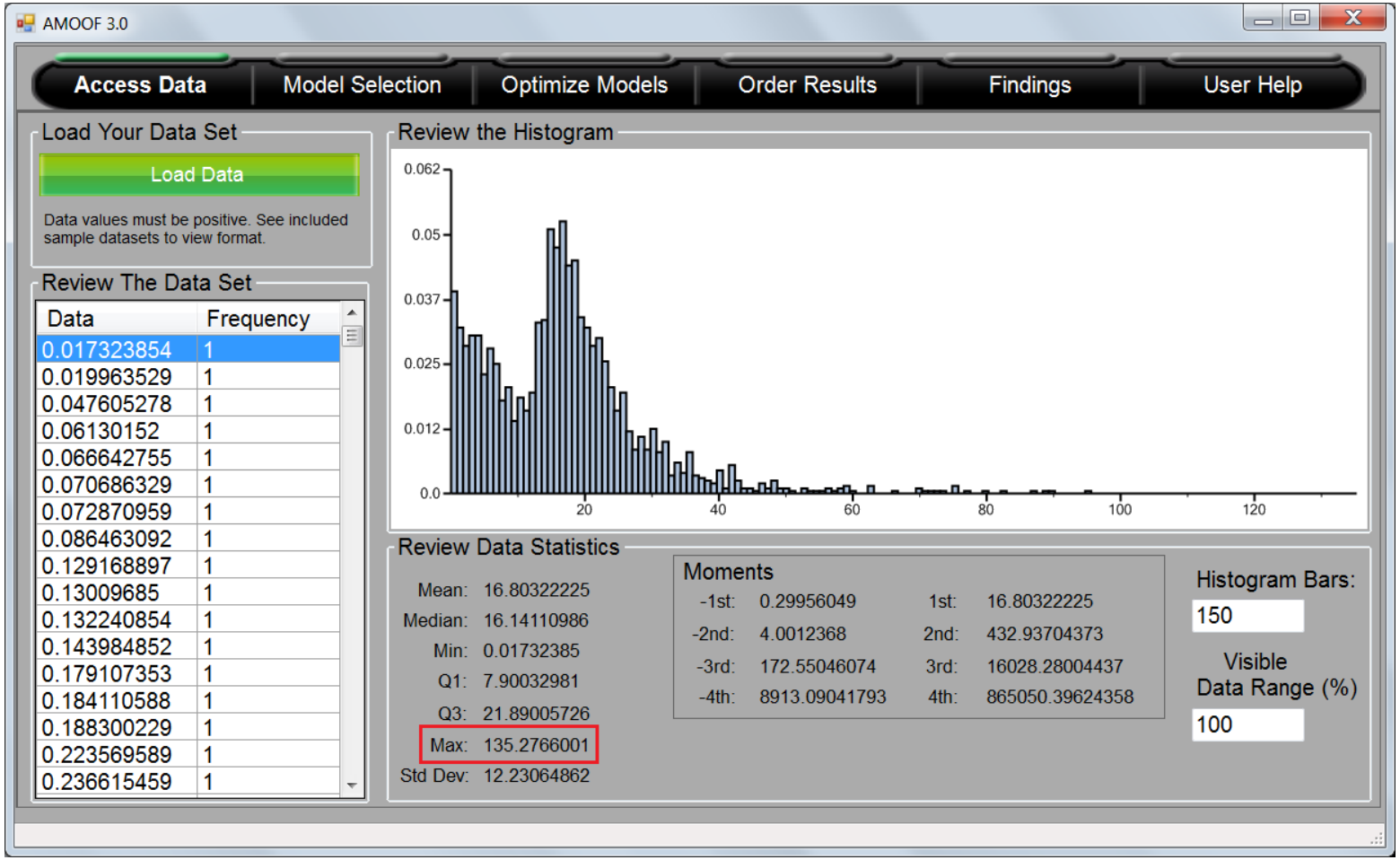
Modify initial parameters



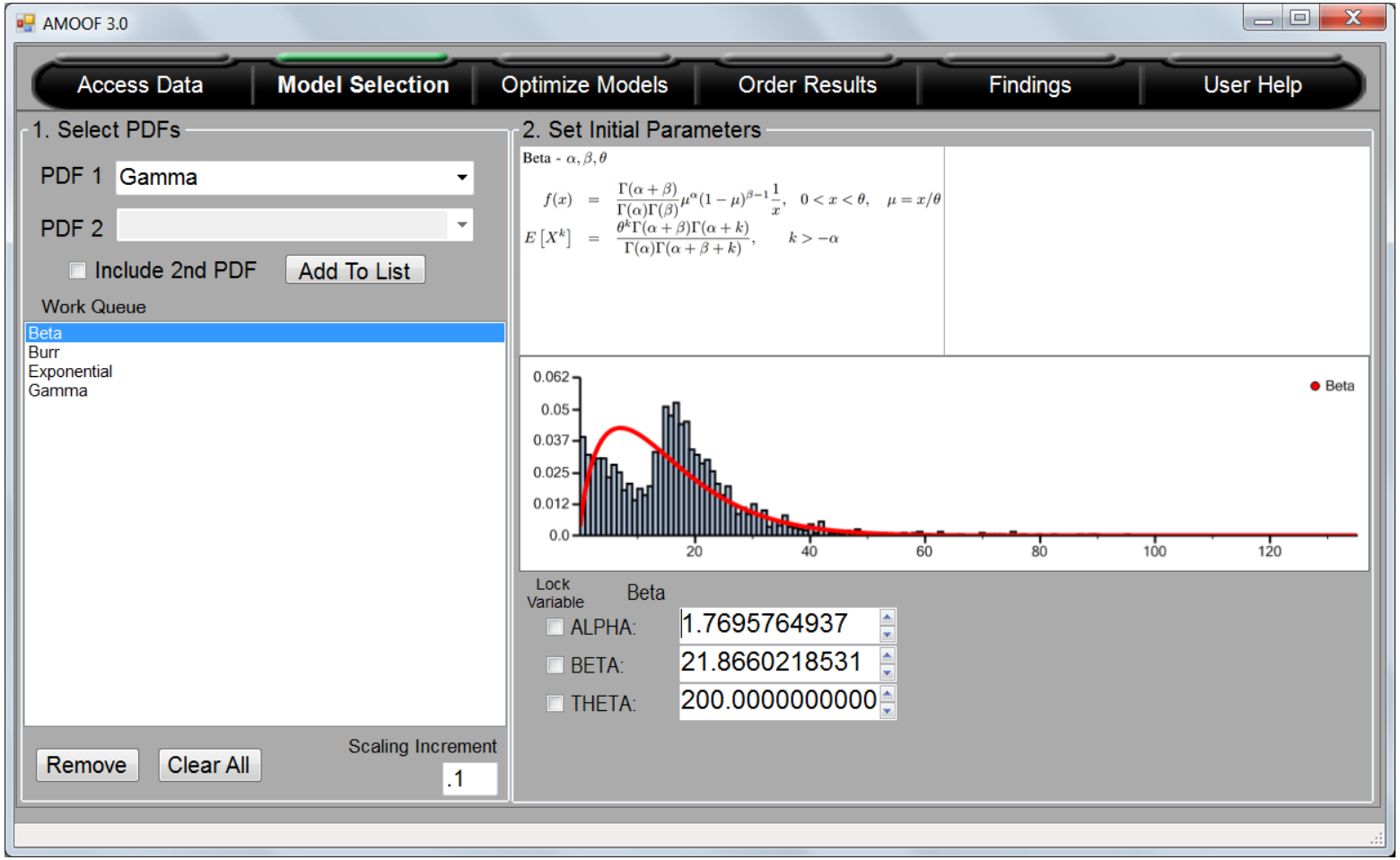
Parameter conditions



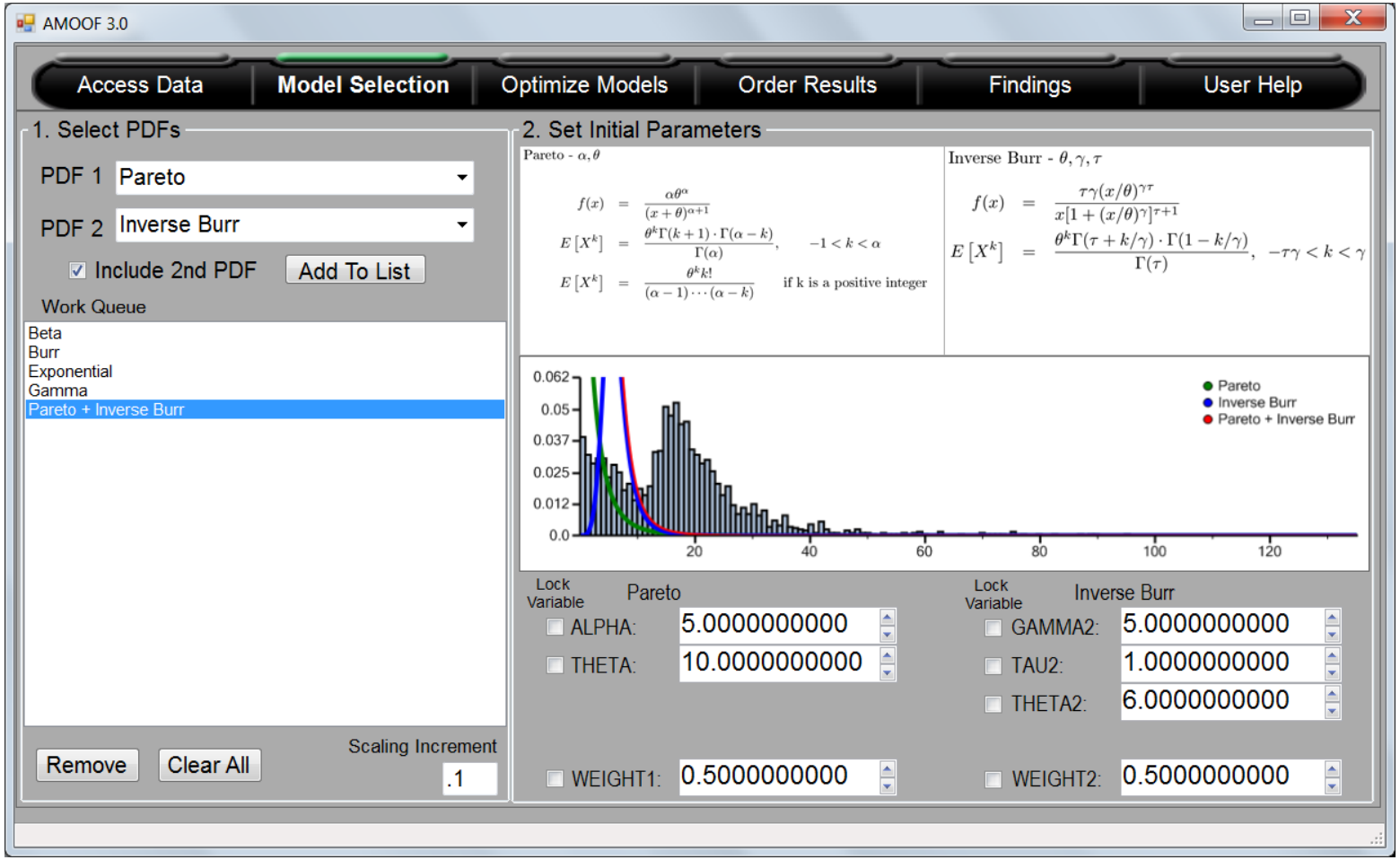
Parameter conditions



Select starting parameters



Mixed pdfs


AMOOF 3.0
Access Data
Model Selection
Optimize Models
Order Results
Findings
User Help
1. Select PDFs
PDF 1
Pareto
PDF 2
Inverse Burr
Include 2nd PDF
Add To List
Work Queue
Beta
Burr
Exponential
Gamma
Pareto + Inverse Burr
Remove
Clear All
Scaling Increment
.1
2. Set Initial Parameters
Pareto - α, θ
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Inverse Burr - θ, γ, τ
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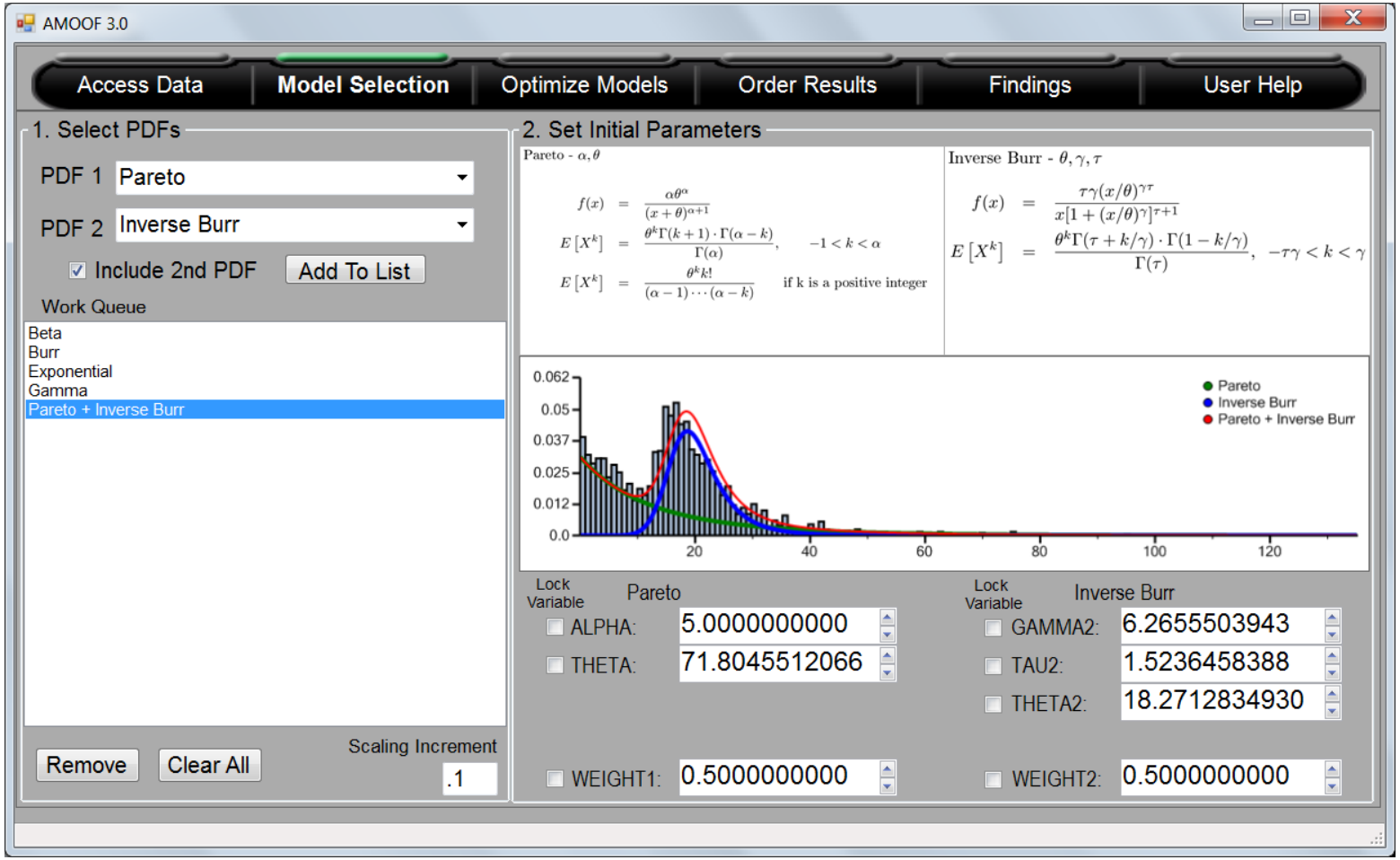
$E[X^k] = \frac{\theta^k\Gamma(\tau+k/\gamma)\cdot\Gamma(1-k/\gamma)}{\Gamma(\tau)}, \quad -\tau\gamma < k < \gamma$
0.062
0.05
0.037
0.025
0.012
0.0
20
40
60
80
100
120
Pareto
Inverse Burr
Pareto + Inverse Burr
Lock Variable
Pareto
ALPHA: 5.0000000000
THETA: 10.0000000000
WEIGHT1: 0.5000000000
Lock Variable
Inverse Burr
GAMMA2: 5.0000000000
TAU2: 1.0000000000
THETA2: 6.0000000000
WEIGHT2: 0.5000000000

Mixed pdfs


AMOOF 3.0
Access Data
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1. Select PDFs
PDF 1
Pareto
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0.062
0.05
0.037
0.025
0.012
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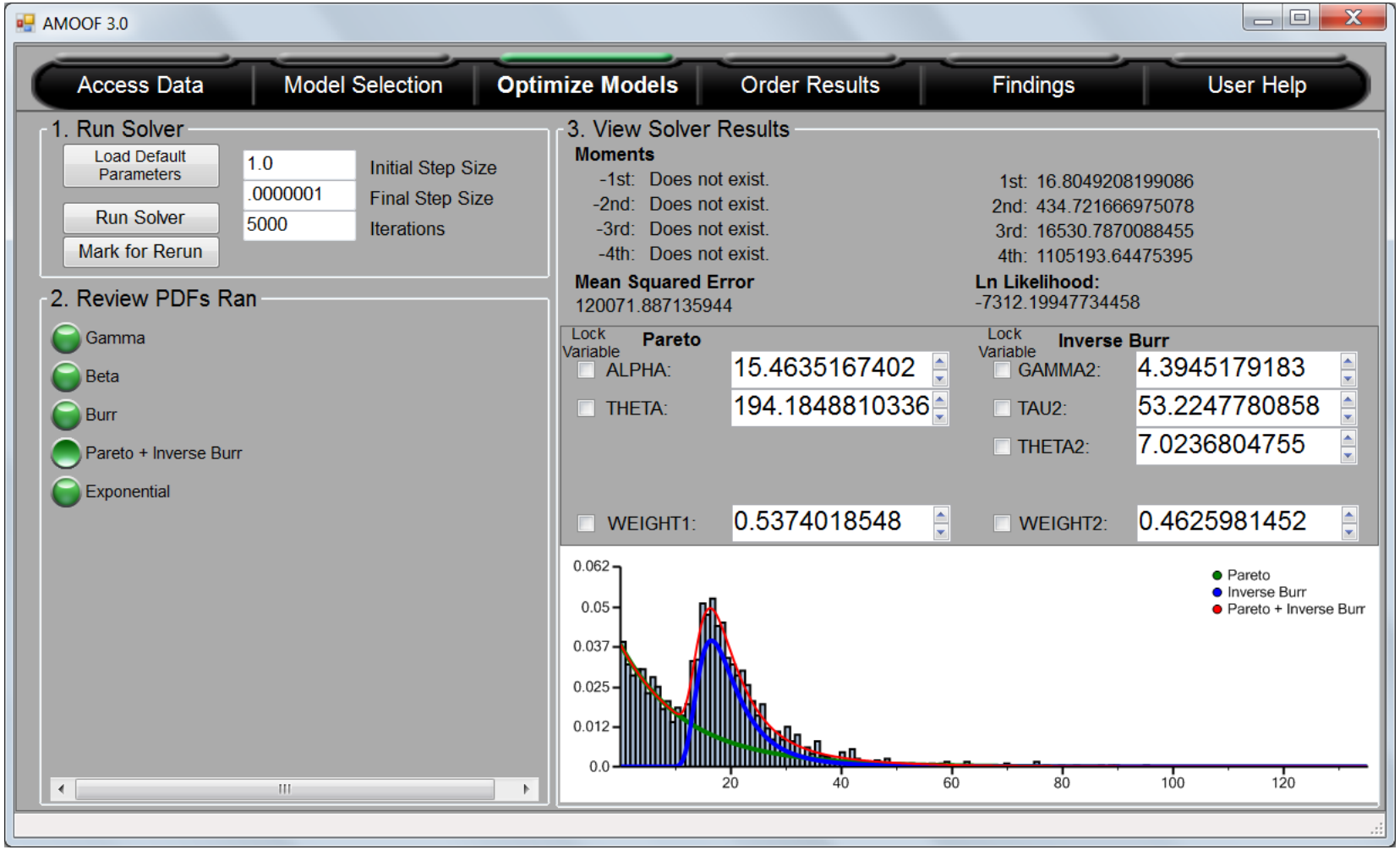
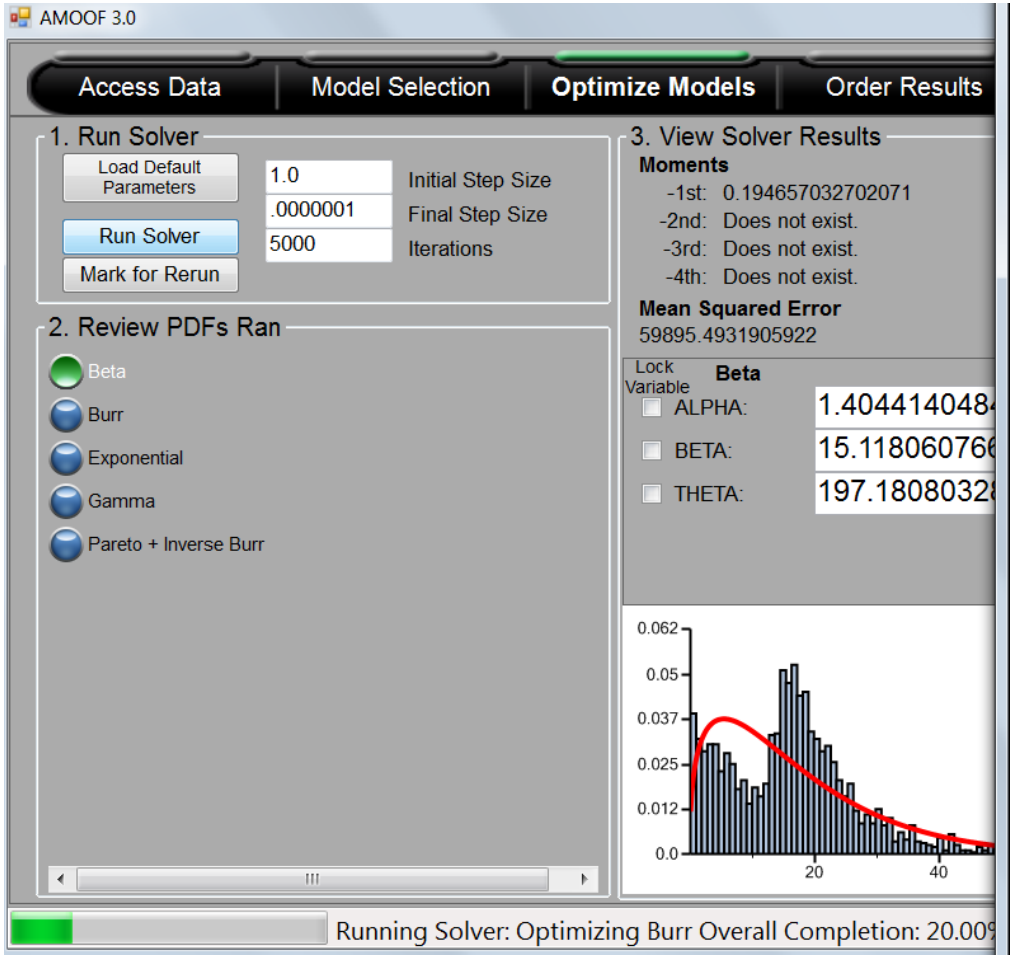
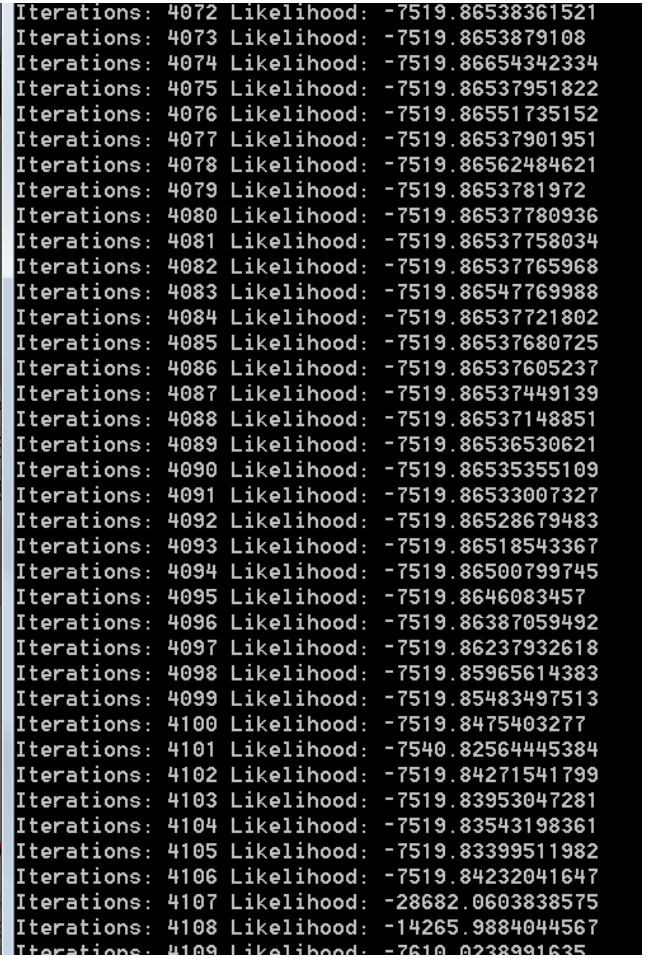
20
40
60
80
100
120
Pareto
Inverse Burr
Pareto + Inverse Burr
Lock Variable
Pareto
ALPHA: 5.0000000000
THETA: 71.8045512066
WEIGHT1: 0.5000000000
Lock Variable
Inverse Burr
GAMMA2: 6.2655503943
TAU2: 1.5236458388
THETA2: 18.2712834930
WEIGHT2: 0.5000000000



OPTIMIZE MODELS PANEL

Run Solver

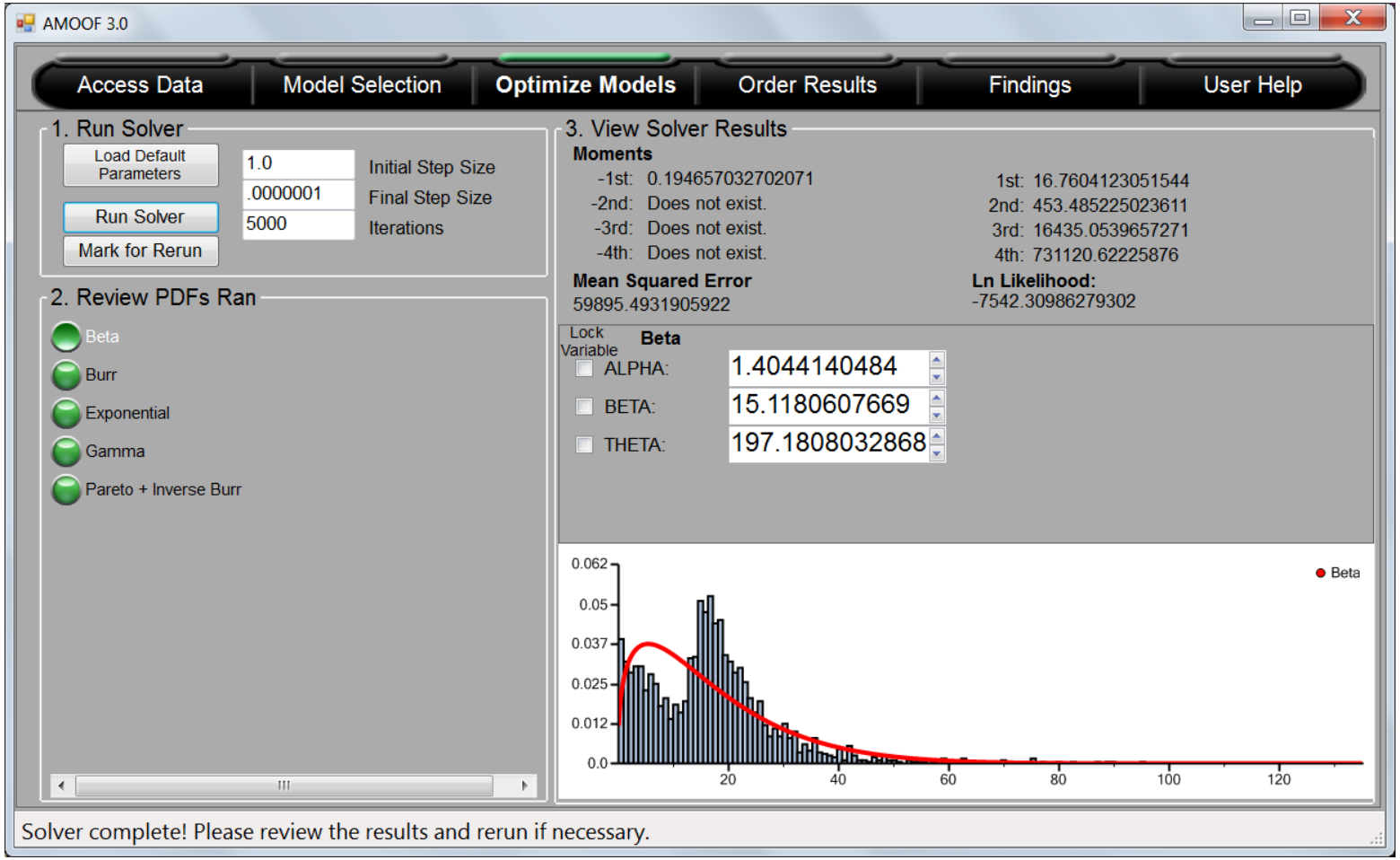

AMOOF 3.0
Access Data
Model Selection
Optimize Models
Order Results
1. Run Solver
Load Default Parameters
1.0
Initial Step Size
.0000001
Final Step Size
Run Solver
5000
Iterations
Mark for Rerun
2. Review PDFs Ran
Beta
Burr
Exponential
Gamma
Pareto + Inverse Burr
3. View Solver Results
Moments
-1st: 0.194657032702071
-2nd: Does not exist.
-3rd: Does not exist.
-4th: Does not exist.
Mean Squared Error
59895.4931905922
Lock Variable
Beta
ALPHA:
BETA:
THETA:
0.062
0.05
0.037
0.025
0.012
0.0
20
40
Running Solver: Optimizing Burr Overall Completion: 20.00%


Iterations: 4072 Likelihood: -7519.86538361521
Iterations: 4073 Likelihood: -7519.8653879108
Iterations: 4074 Likelihood: -7519.86654342334
Iterations: 4075 Likelihood: -7519.86537951822
Iterations: 4076 Likelihood: -7519.86551735152
Iterations: 4077 Likelihood: -7519.86537901951
Iterations: 4078 Likelihood: -7519.86562484621
Iterations: 4079 Likelihood: -7519.8653781972
Iterations: 4080 Likelihood: -7519.86537780936
Iterations: 4081 Likelihood: -7519.86537758034
Iterations: 4082 Likelihood: -7519.86537765968
Iterations: 4083 Likelihood: -7519.86547769988
Iterations: 4084 Likelihood: -7519.86537721802
Iterations: 4085 Likelihood: -7519.86537680725
Iterations: 4086 Likelihood: -7519.86537605237
Iterations: 4087 Likelihood: -7519.86537449139
Iterations: 4088 Likelihood: -7519.86537148851
Iterations: 4089 Likelihood: -7519.86536530621
Iterations: 4090 Likelihood: -7519.86535355109
Iterations: 4091 Likelihood: -7519.86533007327
Iterations: 4092 Likelihood: -7519.86528679483
Iterations: 4093 Likelihood: -7519.86518543367
Iterations: 4094 Likelihood: -7519.86500799745
Iterations: 4095 Likelihood: -7519.8646083457
Iterations: 4096 Likelihood: -7519.86387059492
Iterations: 4097 Likelihood: -7519.86237932618
Iterations: 4098 Likelihood: -7519.85965614383
Iterations: 4099 Likelihood: -7519.85483497513
Iterations: 4100 Likelihood: -7519.8475403277
Iterations: 4101 Likelihood: -7540.82564445384
Iterations: 4102 Likelihood: -7519.84271541799
Iterations: 4103 Likelihood: -7519.83953047281
Iterations: 4104 Likelihood: -7519.83543198361
Iterations: 4105 Likelihood: -7519.83399511982
Iterations: 4106 Likelihood: -7519.84232041647
Iterations: 4107 Likelihood: -28682.0603838575
Iterations: 4108 Likelihood: -14265.9884044567

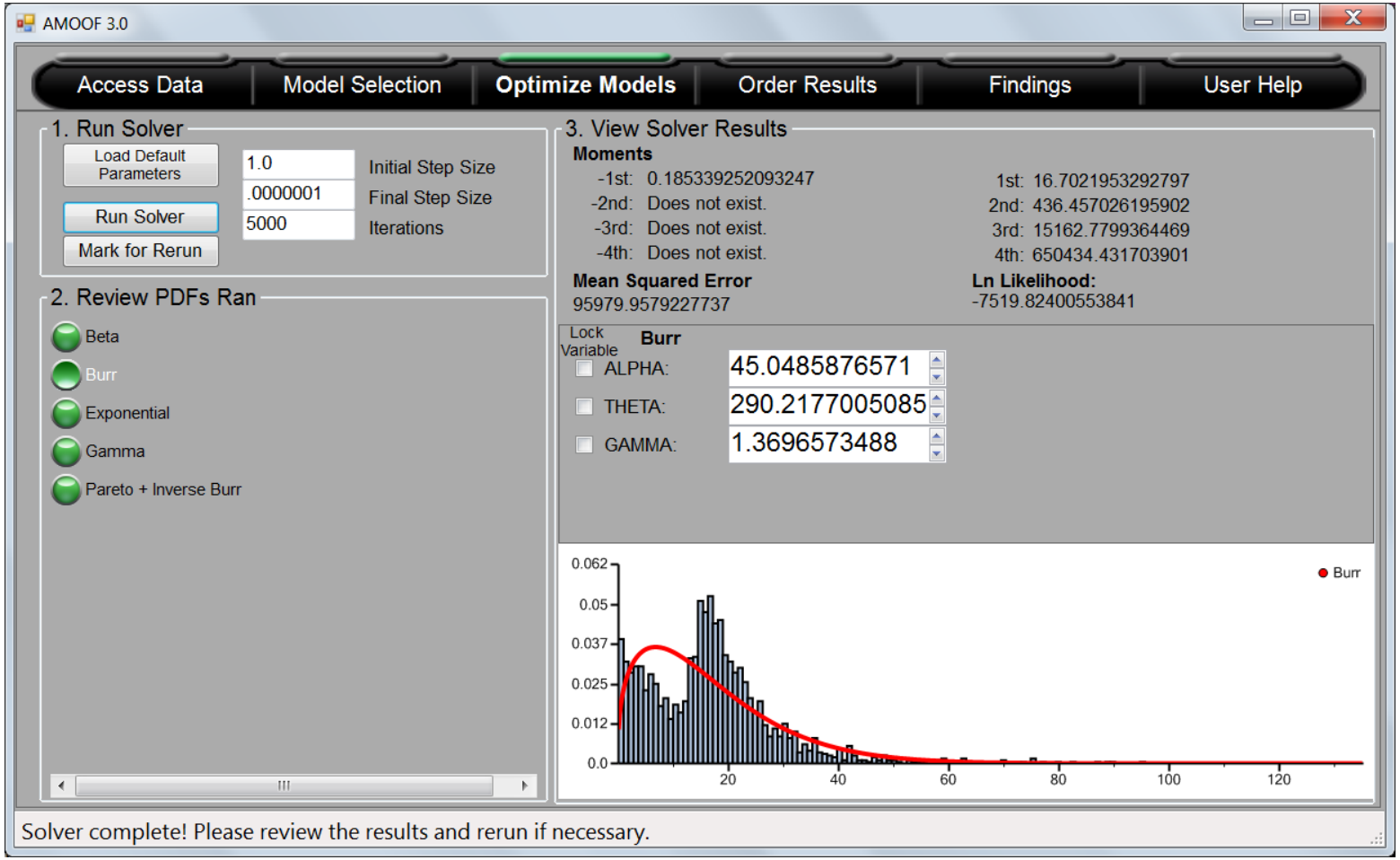
BOBYQA solver

- Bounded Optimization by Quadratic Approximation.
- Released 2009.
- C# port of FORTRAN library
- Uses interpolation points to approximate and maximize the objective function at each iteration.

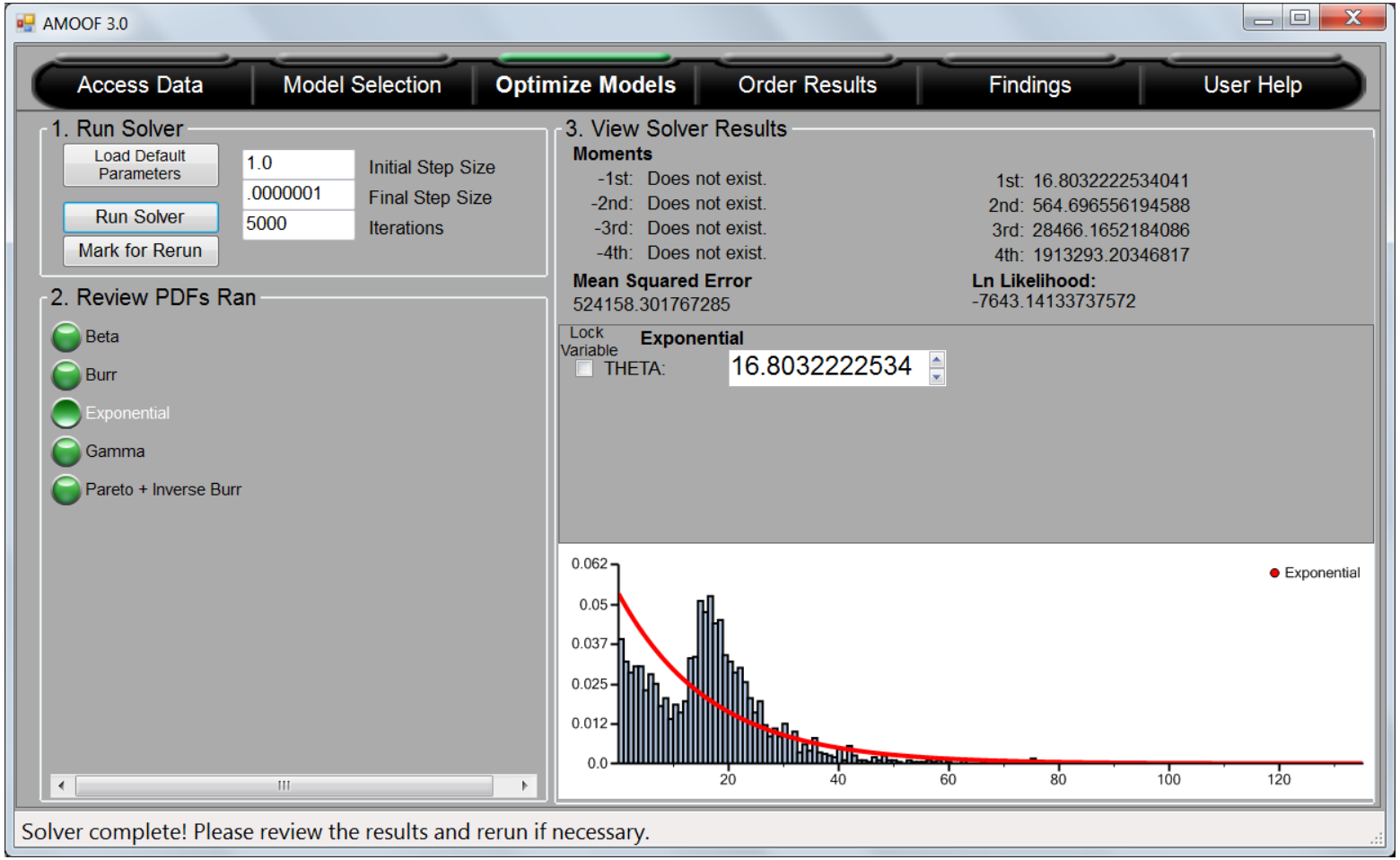
Results of Optimization



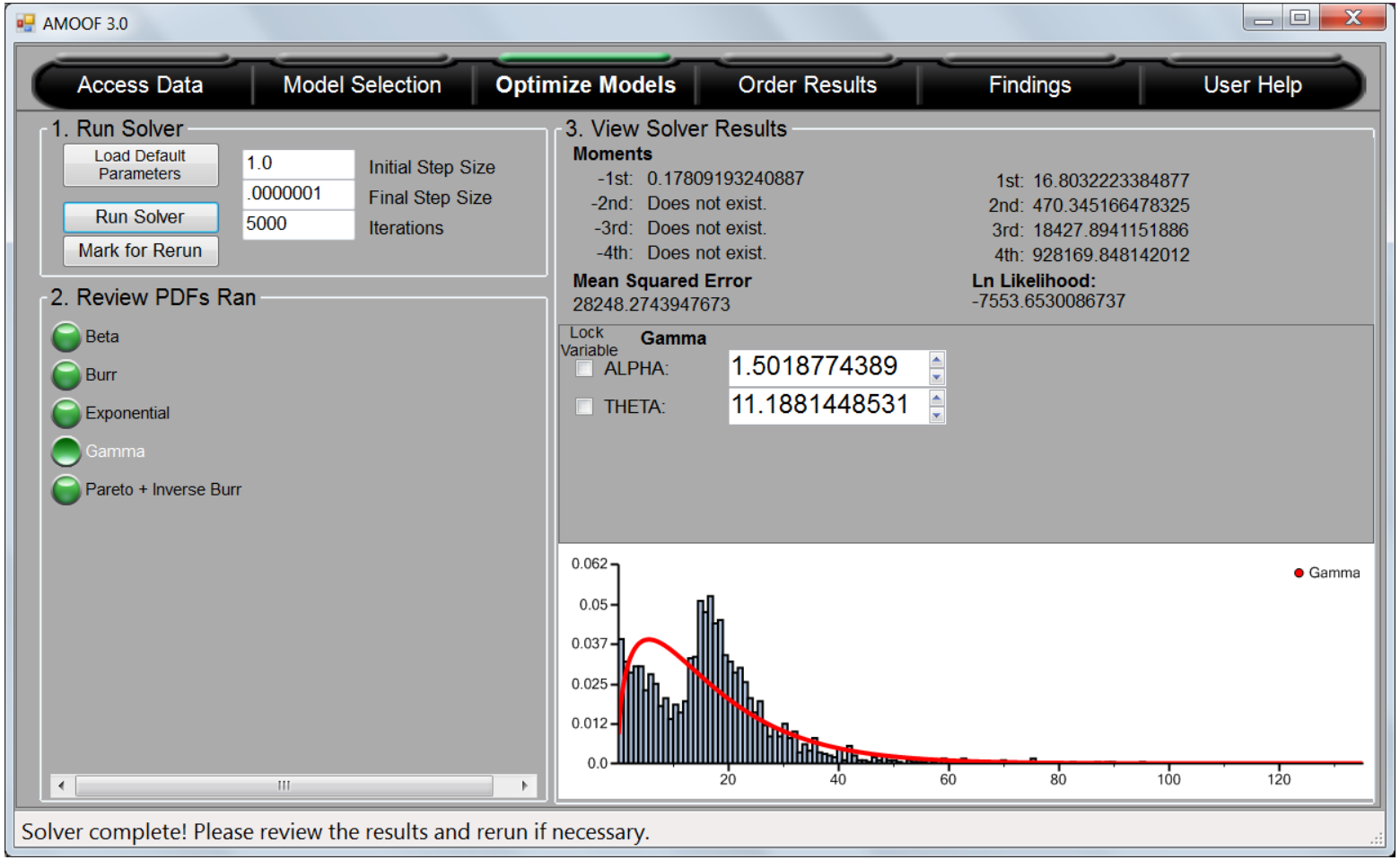
Results of Optimization



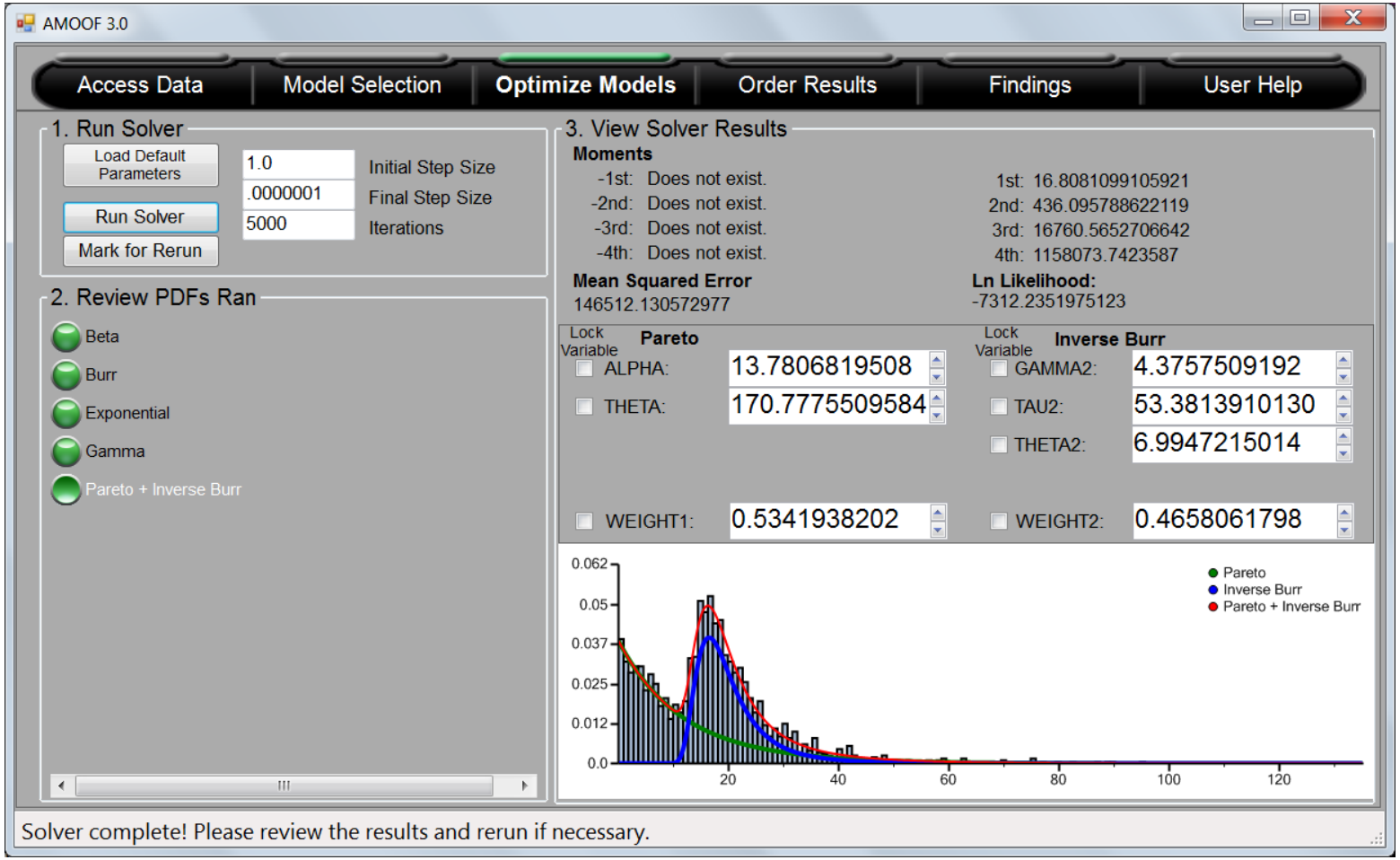
Results of Optimization



Results of Optimization



Results of Optimization

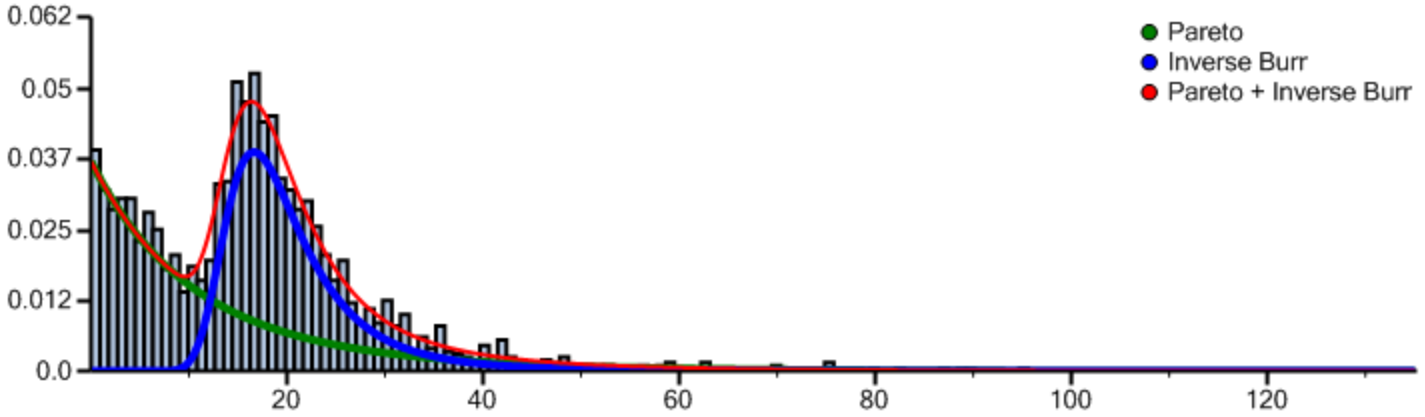


Resulting Log-Likelihood

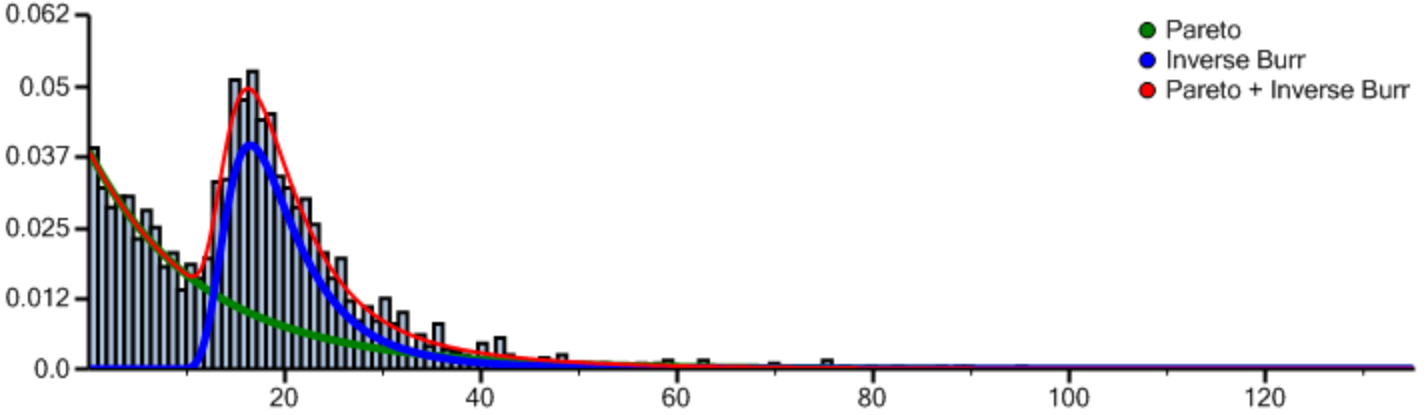
PDF	Before Optimization	After Optimization
Beta	-7576.85	-7542.31
Burr	-27243.86	-7519.82
Exponential	-7965.81	-7643.14
Gamma	-10164.99	-7553.65
Pareto + Inverse Burr	-7371.41	-7312.24

Comparison with Expected Parameters

Expected Curve



Found Curve



Comparison with Expected Parameters

Pareto

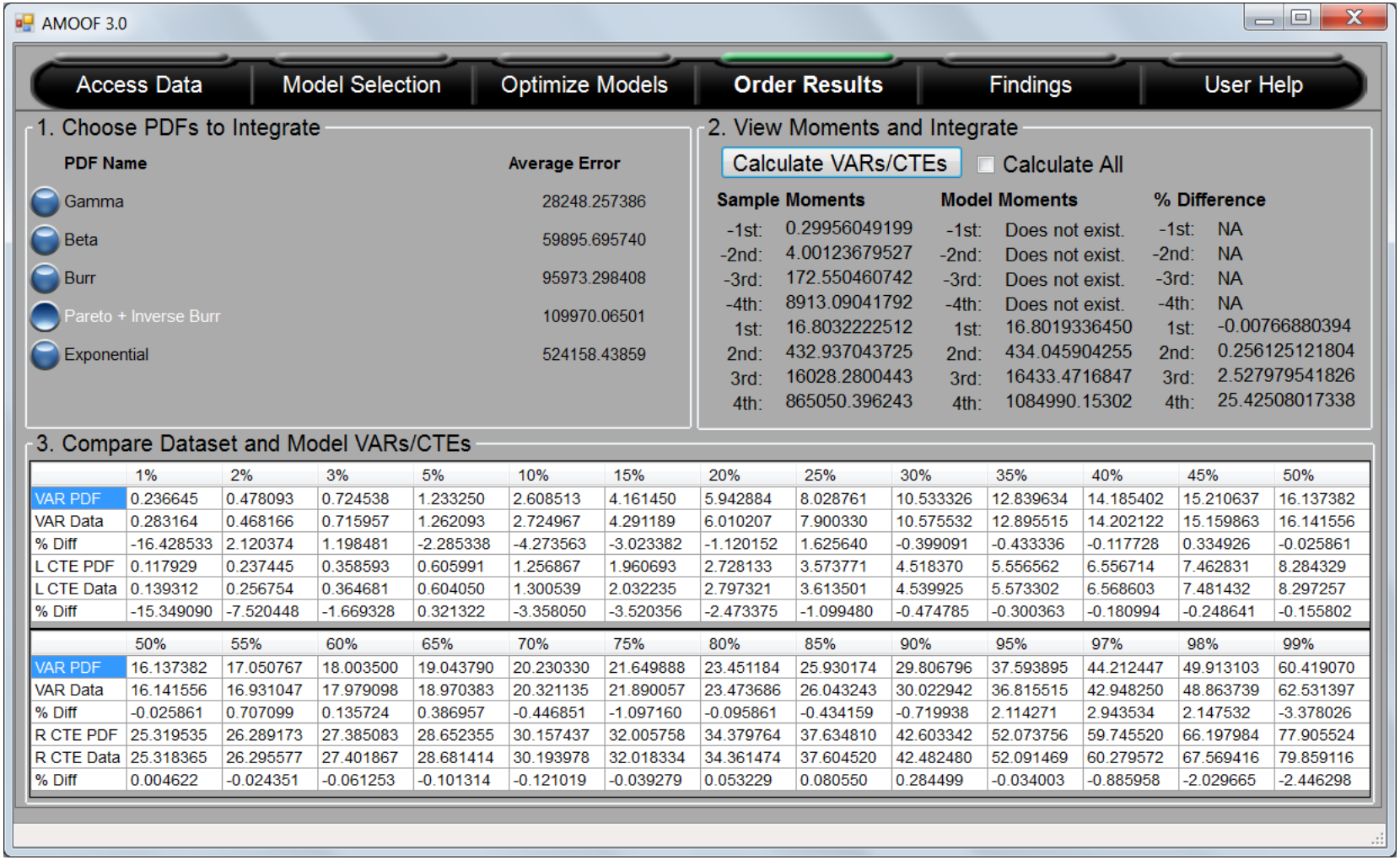
Parameter	Expected Value	Found Value	% Difference
alpha	7.62	13.78	80.85
theta	91.67	170.78	86.30
weight	0.50	0.53	6.84

Inverse Burr

Parameter	Expected value	Found value	% Difference
gamma	4.42	4.38	-1.00
tau	4.83	53.38	1005.20
theta	12.40	6.99	-43.59
weight	0.50	0.47	-6.84

However, the fitted function attains a better log-likelihood than the expected function.

- expected : -7316.92
- found : -7312.24

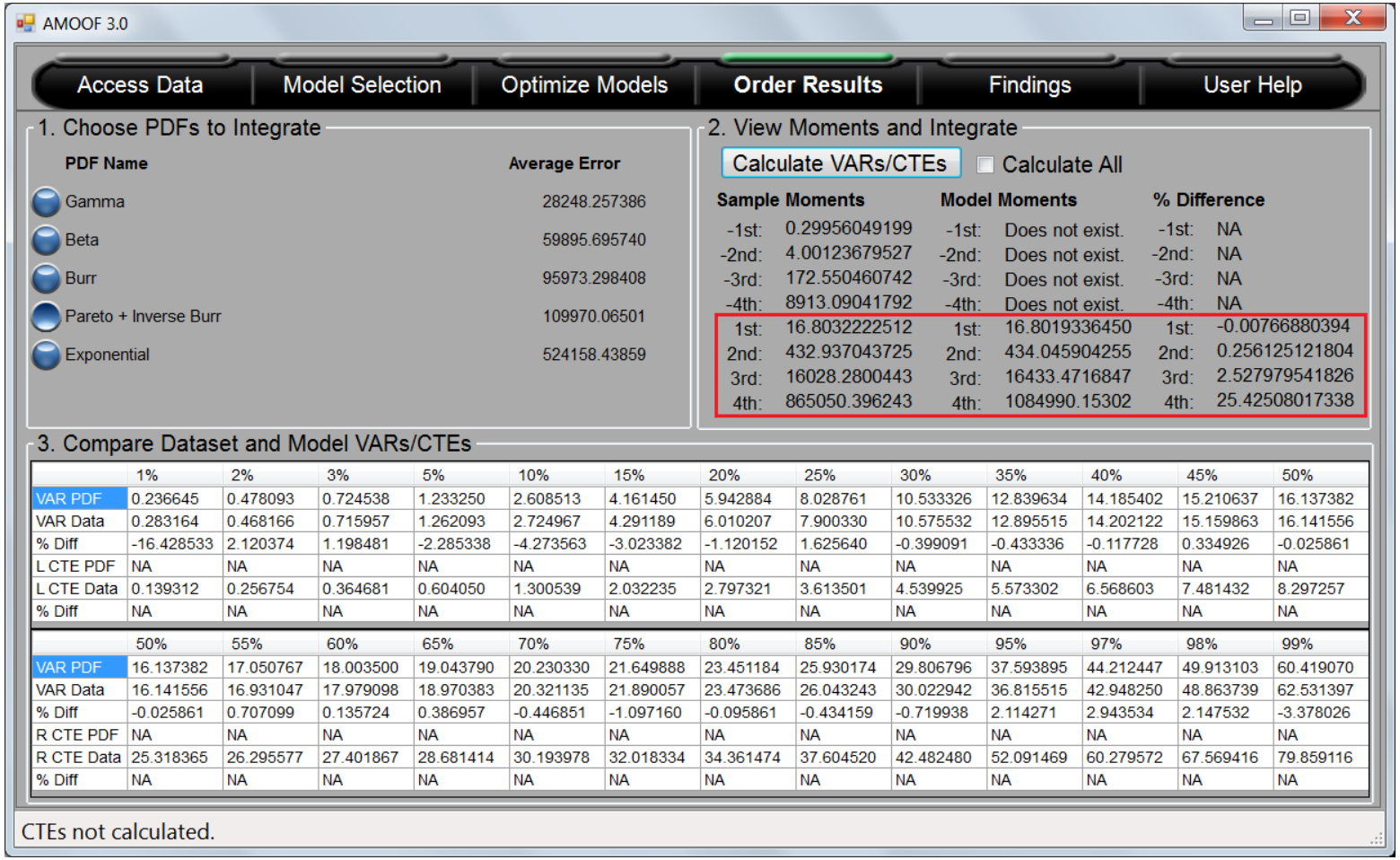


	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.161450	5.942884	8.028761	10.533326	12.839634	14.185402	15.210637	16.137382
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.023382	-1.120152	1.625640	-0.399091	-0.433336	-0.117728	0.334926	-0.025861
L CTE PDF	0.117929	0.237445	0.358593	0.605991	1.256867	1.960693	2.728133	3.573771	4.518370	5.556562	6.556714	7.462831	8.284329
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-15.349090	-7.520448	-1.669328	0.321322	-3.358050	-3.520356	-2.473375	-1.099480	-0.474785	-0.300363	-0.180994	-0.248641	-0.155802

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.649888	23.451184	25.930174	29.806796	37.593895	44.212447	49.913103	60.419070
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.097160	-0.095861	-0.434159	-0.719938	2.114271	2.943534	2.147532	-3.378026
R CTE PDF	25.319535	26.289173	27.385083	28.652355	30.157437	32.005758	34.379764	37.634810	42.603342	52.073756	59.745520	66.197984	77.905524
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	0.004622	-0.024351	-0.061253	-0.101314	-0.121019	-0.039279	0.053229	0.080550	0.284499	-0.034003	-0.885958	-2.029665	-2.446298

ORDER RESULTS PANEL

Compare Moments



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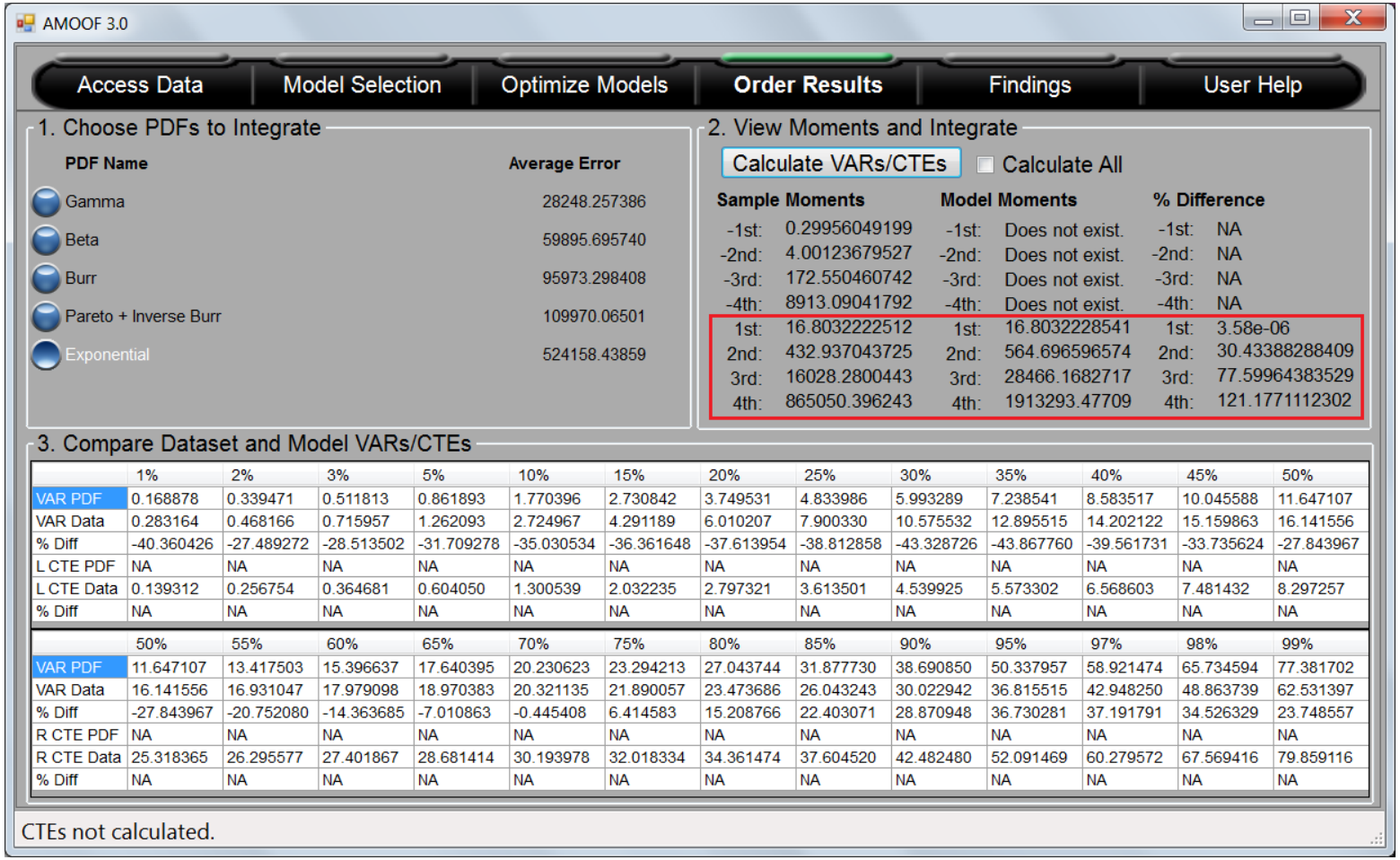
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.161450	5.942884	8.028761	10.533326	12.839634	14.185402	15.210637	16.137382
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.023382	-1.120152	1.625640	-0.399091	-0.433336	-0.117728	0.334926	-0.025861
L CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.649888	23.451184	25.930174	29.806796	37.593895	44.212447	49.913103	60.419070
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.097160	-0.095861	-0.434159	-0.719938	2.114271	2.943534	2.147532	-3.378026
R CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

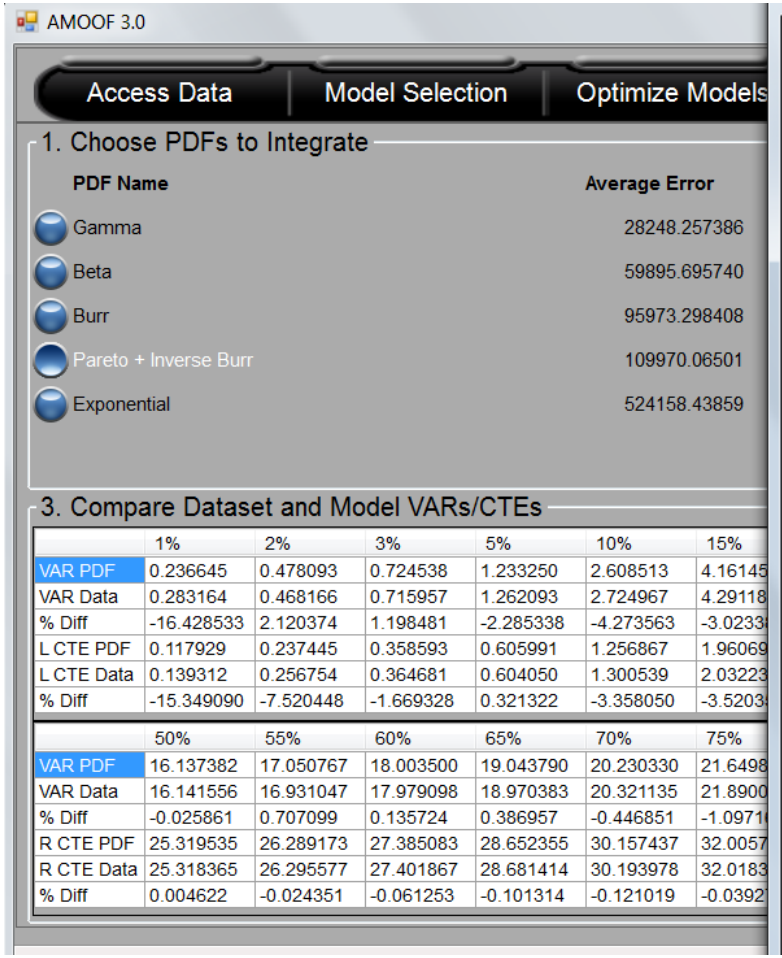
Compare Moments



	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993289	7.238541	8.583517	10.045588	11.647107
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-40.360426	-27.489272	-28.513502	-31.709278	-35.030534	-36.361648	-37.613954	-38.812858	-43.328726	-43.867760	-39.561731	-33.735624	-27.843967
L CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	11.647107	13.417503	15.396637	17.640395	20.230623	23.294213	27.043744	31.877730	38.690850	50.337957	58.921474	65.734594	77.381702
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-27.843967	-20.752080	-14.363685	-7.010863	-0.445408	6.414583	15.208766	22.403071	28.870948	36.730281	37.191791	34.526329	23.748557
R CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Calculate CTEs



	1%	2%	3%	5%	10%	15%
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.16145
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.29118
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.0233
L CTE PDF	0.117929	0.237445	0.358593	0.605991	1.256867	1.96069
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.03223
% Diff	-15.349090	-7.520448	-1.669328	0.321322	-3.358050	-3.5203

	50%	55%	60%	65%	70%	75%
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.6498
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.8900
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.0971
R CTE PDF	25.319535	26.289173	27.385083	28.652355	30.157437	32.0057
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.0183
% Diff	0.004622	-0.024351	-0.061253	-0.101314	-0.121019	-0.0392

```
 *** allocatemem: Warning: doubling stack size; new stack = 64000000 (
  realprecision = 105 significant digits (100 digits displayed)


----------entering calculateAllVARCTEs
Function Component 1:
0.538371431825554×Pareto_(xVar,16.2034719757771,204.395074453393)

Function Component 2:
0.461628568174446×InverseBurr_(xVar,4.40223310128007,50.3932108251082,7

PDF Subcomponent Areas:
Expected: 0.538371431 Found: 0.538371430 Error: 0.00000000260362495
Expected: 0.461628568 Found: 0.461628567 Error: 0.00000000234239567

TOTAL AREA: 0.9999999975169659
Bounds:
[0.00000002007348633    529.4078434]
[8.335383893    1913.551952]
calculate VARCTE. Expected Area: 0.01000000000 Found Area: 0.0100000000
calculate VARCTE. Expected Area: 0.02000000000 Found Area: 0.0200000000
calculate VARCTE. Expected Area: 0.03000000000 Found Area: 0.0300000000
calculate VARCTE. Expected Area: 0.05000000000 Found Area: 0.0500000000
calculate VARCTE. Expected Area: 0.1000000000 Found Area: 0.1000000000
calculate VARCTE. Expected Area: 0.1500000000 Found Area: 0.1500000000
calculate VARCTE. Expected Area: 0.2000000000 Found Area: 0.2000000000
calculate VARCTE. Expected Area: 0.2500000000 Found Area: 0.2500000000
calculate VARCTE. Expected Area: 0.3000000000 Found Area: 0.3000000000
calculate VARCTE. Expected Area: 0.3500000000 Found Area: 0.3500000000
calculate VARCTE. Expected Area: 0.4000000000 Found Area: 0.4000000000
calculate VARCTE. Expected Area: 0.4500000000 Found Area: 0.4500000000
calculate VARCTE. Expected Area: 0.5000000000 Found Area: 0.5000000000
calculate VARCTE. Expected Area: 0.5500000000 Found Area: 0.5500000000
calculate VARCTE. Expected Area: 0.6000000000 Found Area: 0.6000000000
calculate VARCTE. Expected Area: 0.6500000000 Found Area: 0.6500000000
calculate VARCTE. Expected Area: 0.7000000000 Found Area: 0.7000000000
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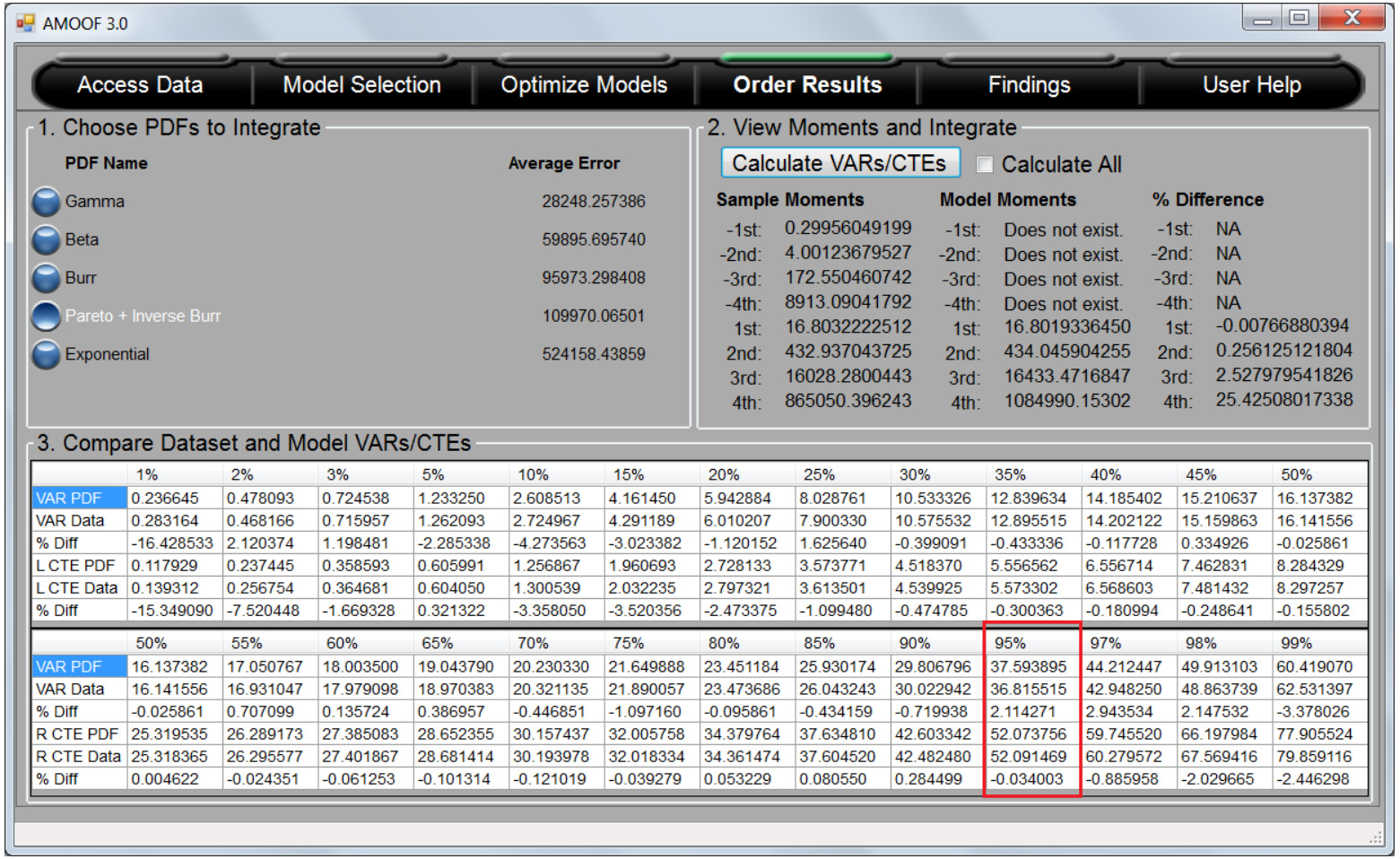
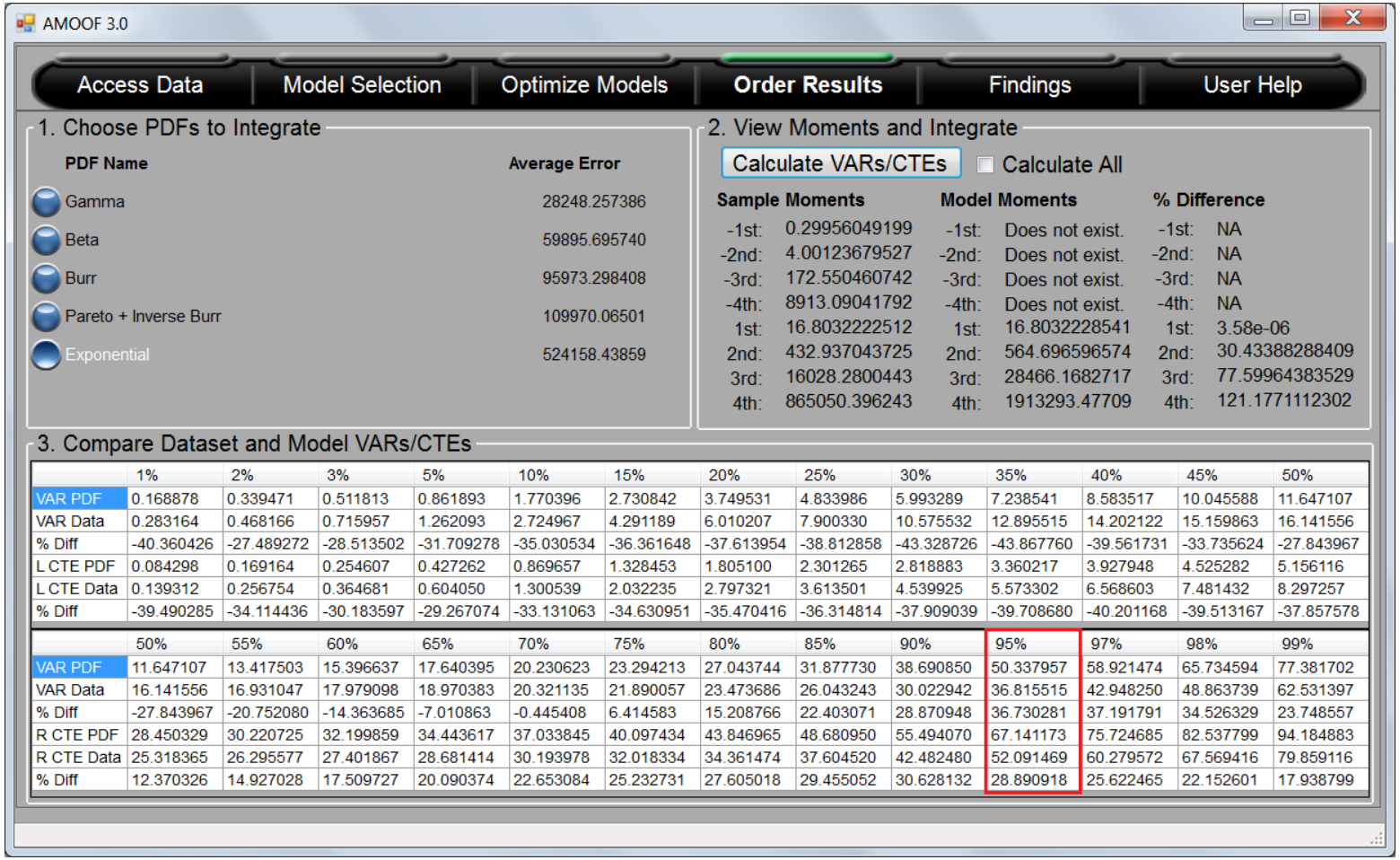
Compare VaRs/CTEs



	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.161450	5.942884	8.028761	10.533326	12.839634	14.185402	15.210637	16.137382
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.023382	-1.120152	1.625640	-0.399091	-0.433336	-0.117728	0.334926	-0.025861
L CTE PDF	0.117929	0.237445	0.358593	0.605991	1.256867	1.960693	2.728133	3.573771	4.518370	5.556562	6.556714	7.462831	8.284329
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-15.349090	-7.520448	-1.669328	0.321322	-3.358050	-3.520356	-2.473375	-1.099480	-0.474785	-0.300363	-0.180994	-0.248641	-0.155802

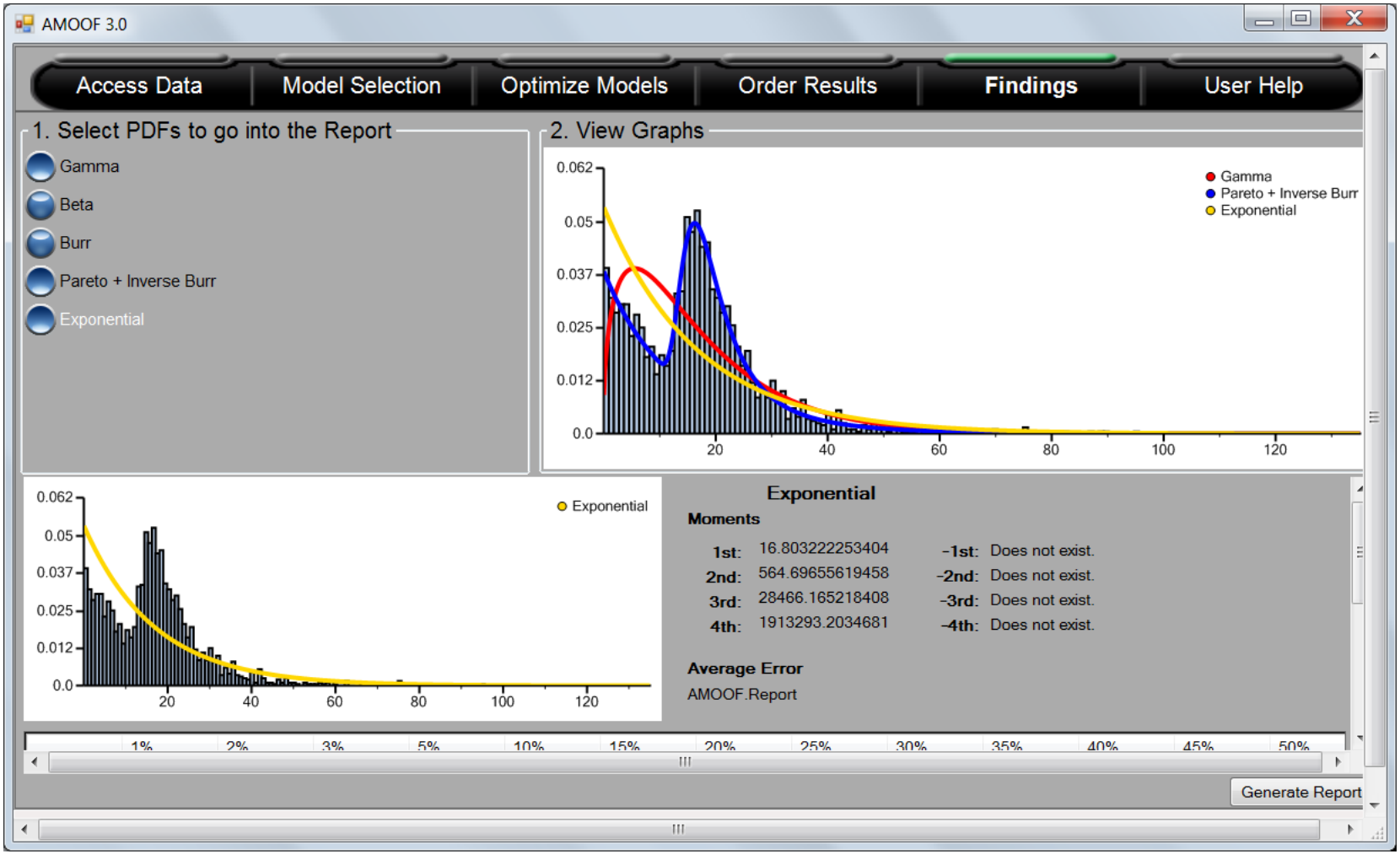
	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.649888	23.451184	25.930174	29.806796	37.593895	44.212447	49.913103	60.419070
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.097160	-0.095861	-0.434159	-0.719938	2.114271	2.943534	2.147532	-3.378026
R CTE PDF	25.319535	26.289173	27.385083	28.652355	30.157437	32.005758	34.379764	37.634810	42.603342	52.073756	59.745520	66.197984	77.905524
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	0.004622	-0.024351	-0.061253	-0.101314	-0.121019	-0.039279	0.053229	0.080550	0.284499	-0.034003	-0.885958	-2.029665	-2.446298

Compare VaRs/CTEs



	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993289	7.238541	8.583517	10.045588	11.647107
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-40.360426	-27.489272	-28.513502	-31.709278	-35.030534	-36.361648	-37.613954	-38.812858	-43.328726	-43.867760	-39.561731	-33.735624	-27.843967
L CTE PDF	0.084298	0.169164	0.254607	0.427262	0.869657	1.328453	1.805100	2.301265	2.818883	3.360217	3.927948	4.525282	5.156116
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-39.490285	-34.114436	-30.183597	-29.267074	-33.131063	-34.630951	-35.470416	-36.314814	-37.909039	-39.708680	-40.201168	-39.513167	-37.857578

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	11.647107	13.417503	15.396637	17.640395	20.230623	23.294213	27.043744	31.877730	38.690850	50.337957	58.921474	65.734594	77.381702
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-27.843967	-20.752080	-14.363685	-7.010863	-0.445408	6.414583	15.208766	22.403071	28.870948	36.730281	37.191791	34.526329	23.748557
R CTE PDF	28.450329	30.220725	32.199859	34.443617	37.033845	40.097434	43.846965	48.680950	55.494070	67.141173	75.724685	82.537799	94.184883
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	12.370326	14.927028	17.509727	20.090374	22.653084	25.232731	27.605018	29.455052	30.628132	28.890918	25.622465	22.152601	17.938799



FINDINGS PANEL

AMOOF Risk Analysis Report

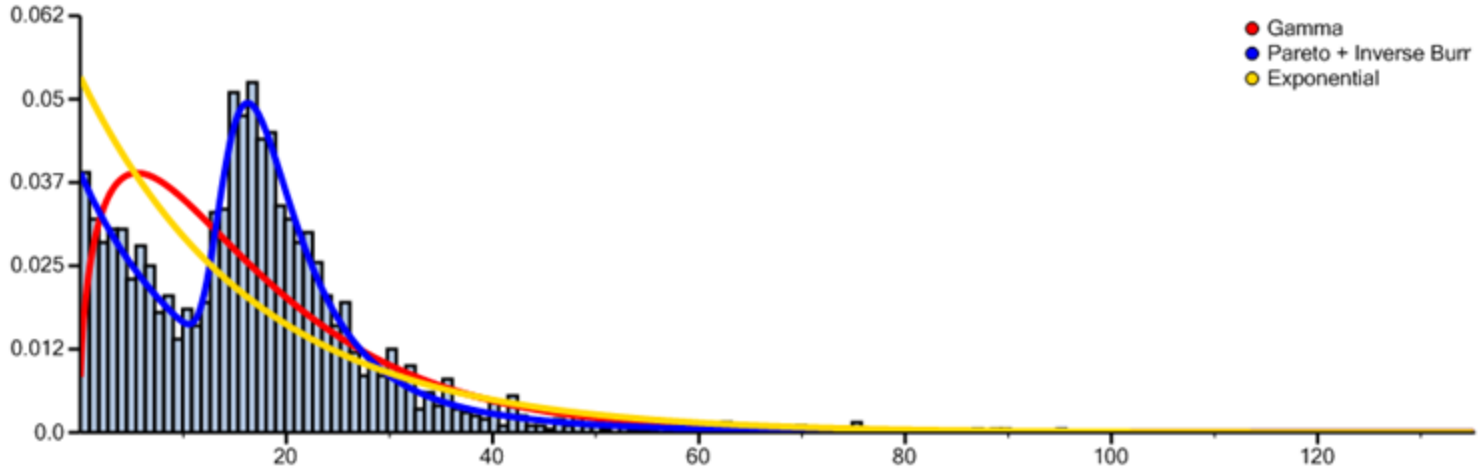
7/9/2014 3:01:35 PM

Report generated by AMOOF 3.0

Data File: Pareto and Inverse Burr 2.csv

Dataset Statistics		Dataset Moments	
Mean	16.803222251257	-1st	0.299560491996276
Median	16.14110986	-2nd	4.00123679527133
Q1	7.900329808	-3rd	172.550460742789
Q3	21.89005726	-4th	8913.09041792718
Min	0.017323854	1st	16.803222251257
Max	135.2766001	2nd	432.937043725345
Std Dev	12.2306486214022	3rd	16028.2800443683
Count	2000	4th	865050.396243577

PDFs



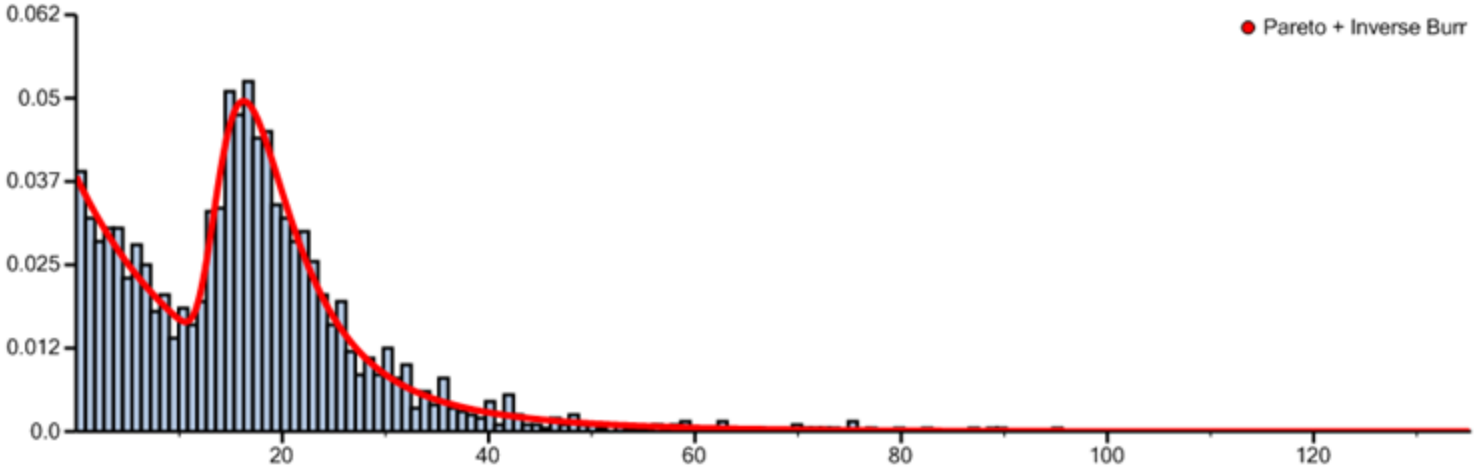
Pareto + Inverse Burr

Moment RMSE (existing moments only): 120071.887135944

Log-Likelihood: -7312.19947734458

Kolmogorov Smirnov Test: 0.009499803794918 16

Anderson-Darling Test: 0.153311788554447



Moments		
Moment	Data	Model
-1st	0.299560491996276	Does not exist.
-2nd	4.00123679527133	Does not exist.
-3rd	172.550460742789	Does not exist.
-4th	8913.09041792718	Does not exist.
1st	16.803222251257	16.8049208199086
2nd	432.937043725345	434.721666975078
3rd	16028.2800443683	16530.7870088455
4th	865050.396243577	1105193.64475395

Parameters	
ALPHA	15.4635167401687
THETA	194.184881033639
GAMMA2	4.39451791834518
TAU2	53.2247780858357
THETA2	7.0236804754546
WEIGHT1	0.537401854797764
WEIGHT2	0.462598145202236

Conditional Tail Expectations and Value at Risks:

	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.236017	0.476848	0.722687	1.230222	2.602825	4.153703	5.934075	8.020584	10.528928	12.839738	14.185746	15.211064	16.137973
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-16.650010	1.854527	0.939890	-2.525274	-4.482330	-3.203915	-1.266721	1.522142	-0.440672	-0.432525	-0.115310	0.337741	-0.022197
LCTE PDF	0.117615	0.236819	0.357660	0.604454	1.253899	1.956454	2.722858	3.567816	4.512296	5.551140	6.552005	7.458687	8.280650
LCTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-15.574692	-7.764062	-1.925254	0.066768	-3.586220	-3.728932	-2.661949	-1.264285	-0.608565	-0.397651	-0.252686	-0.304029	-0.200144

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	16.137973	17.051582	18.004571	19.045122	20.231898	21.651619	23.452934	25.931758	29.808479	37.602189	44.236835	49.959690	60.525372
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-0.022197	0.711912	0.141678	0.393981	-0.439133	-1.089255	-0.088403	-0.428078	-0.714330	2.136800	3.000319	2.242872	-3.208028
RCTE PDF	25.329188	26.299822	27.396944	28.665740	30.172810	32.023874	34.401970	37.663858	42.646142	52.155781	59.872457	66.371393	78.181652
RCTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	0.042751	0.016143	-0.017965	-0.054647	-0.070105	0.017301	0.117853	0.157796	0.385245	0.123459	-0.675378	-1.773026	-2.100529

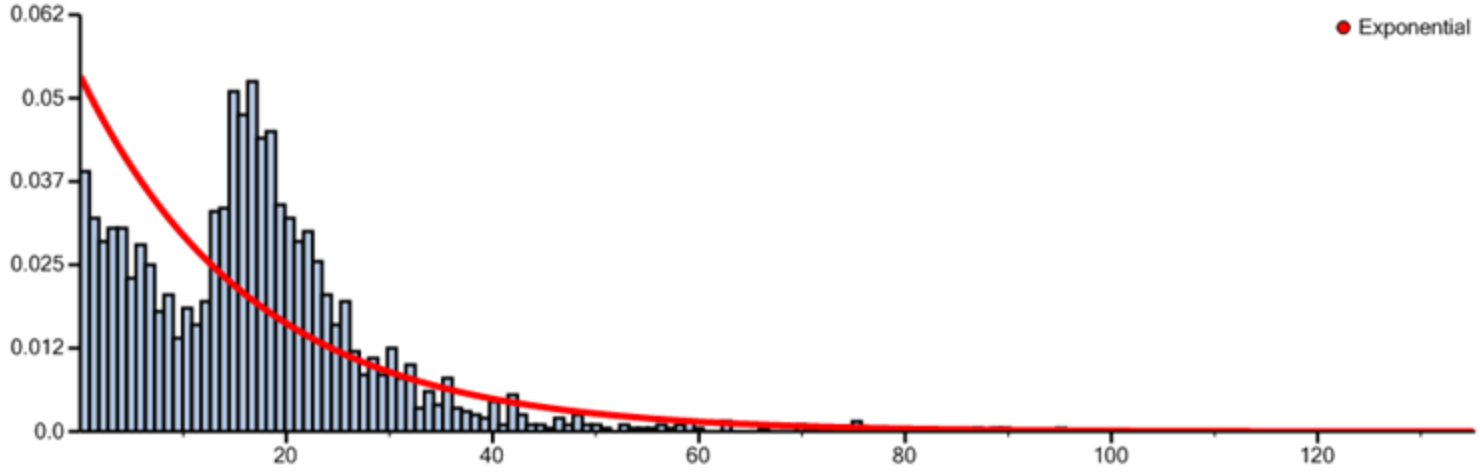
Exponential

Moment RMSE (existing moments only): 524158.301767285

Log-Likelihood: -7643.14133737572

Kolmogorov Smirnov Test: 0.187578661893205

Anderson-Darling Test: 87.5504631251774



Moments		
Moment	Data	Model
-1st	0.299560491996276	Does not exist.
-2nd	4.00123679527133	Does not exist.
-3rd	172.550460742789	Does not exist.
-4th	8913.09041792718	Does not exist.
1st	16.803222251257	16.8032222534041
2nd	432.937043725345	564.696556194588
3rd	16028.2800443683	28466.1652184086
4th	865050.396243577	1913293.20346817

Parameters	
THETA	16.8032222534041

Conditional Tail Expectations and Value at Risks:

	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993288	7.238541	8.583517	10.045588	11.647106
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-40.360426	-27.489272	-28.513502	-31.709282	-35.030536	-36.361650	-37.613956	-38.812861	-43.328728	-43.867762	-39.561733	-33.735627	-27.843970
LCTE PDF	0.084298	0.169164	0.254607	0.427262	0.869657	1.328453	1.805099	2.301265	2.818883	3.360217	3.927948	4.525281	5.156116
LCTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-39.490283	-34.114434	-30.183594	-29.267080	-33.131065	-34.630953	-35.470419	-36.314816	-37.909042	-39.708682	-40.201170	-39.513169	-37.857581

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	11.647106	13.417502	15.396637	17.640394	20.230623	23.294212	27.043743	31.877729	38.690849	50.337955	58.921472	65.734592	77.381698
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-27.843970	-20.752083	-14.363688	-7.010867	-0.445411	6.414579	15.208762	22.403067	28.870944	36.730275	37.191786	34.526325	23.748551
RCTE PDF	28.450328	30.220724	32.199858	34.443616	37.033844	40.097433	43.846963	48.680949	55.494067	67.141171	75.724682	82.537795	94.184885
RCTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	12.370322	14.927023	17.509723	20.090370	22.653079	25.232727	27.605013	29.455047	30.628126	28.890915	25.622461	22.152595	17.938802

Help and Downloads

- Website: https://bitbucket.org/AMOOF3/amoof-3.0/wiki/Home
- Download:
- https://bitbucket.org/AMOOF3/amoof-3.0/downloads

AMOOF3: Stochastic Efficient Modeling Application

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Joint work between the University of Illinois at
Urbana-Champaign (UIUC) and Central Washington University (CWU)
Funded by The Actuarial Foundation

Purpose

- Used AMOOF3 (Actuarial Model Optimal Outcome Fit V3.0) to analyze statutory ending surplus data from a real block of universal life insurance, provided by Milliman, i.e. “Milliman data”

Milliman Data

- Present value of ending surplus data at 30 years (360 months), i.e, “ending surplus,” was the output from a real block of universal life insurance using a proprietary stochastic scenario generator
- 50,000 stochastic economic 7-year US treasury yield scenarios were considered, where each scenario is a random path of monthly portfolio yield rates $x = (r_1, r_2, \ldots, r_{360})$
- We called the 50,000 ending surplus data the “full run distribution,” (true distribution)

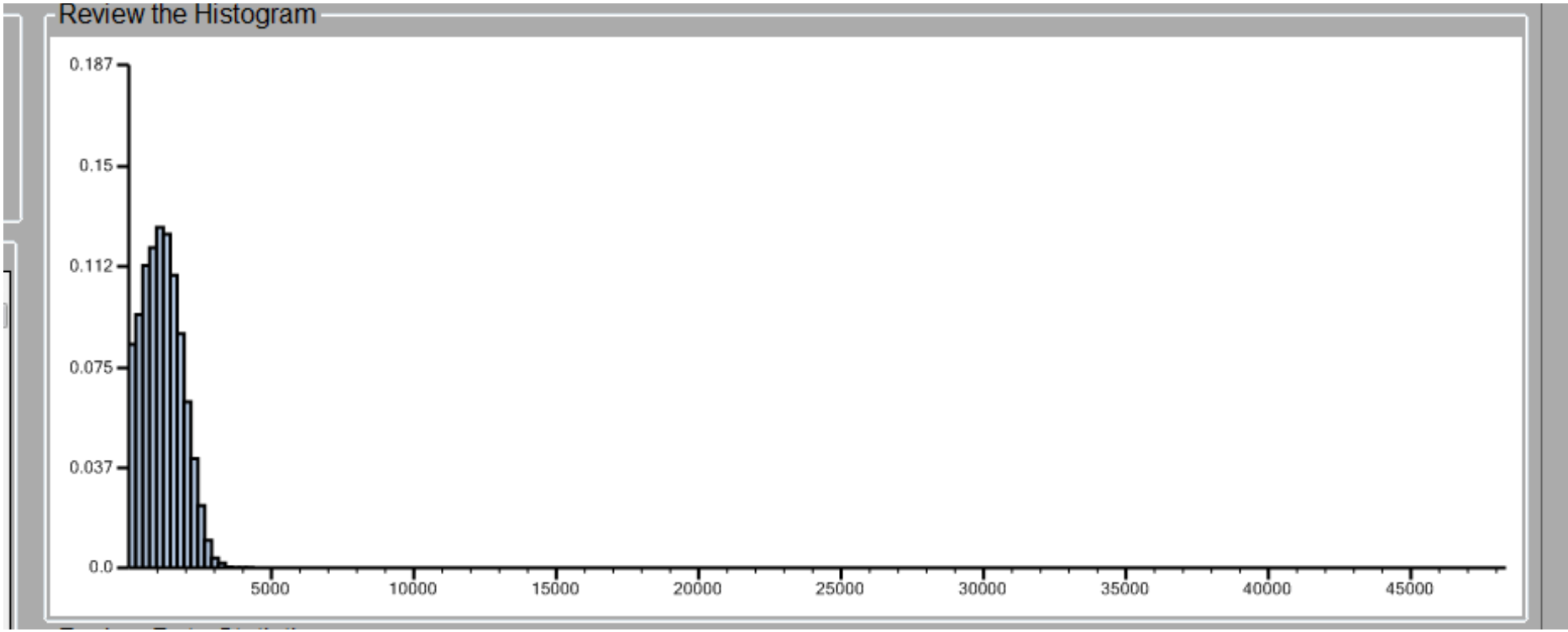
Data Transformation

- The full run distribution was transformed by:
  - Dividing all ending surplus values by 1000
  - Muliplying all ending surplus values by negative one
  - Ignoring any negative values, corresponding to a positive ending surplus value
- Therefore, we only focused on the tail distribution for ending surplus (the worst ending surplus values, 38,137 of the original 50,000 observations)
- The resulting distribution was the “transformed full run distribution,” and various statistics were accurately and efficiently computed using AMOOF3

Trans Full Run Distribution: Statistics

Statistic Name	Statistic Value
Mean	1172.79
Median	1135.62
Standard Deviation	714.51
Minimum	0.14
Maximum	48,341.36
CTE70	1993.12
CTE90	2420.63
CTE99	3122.65

Trans Full Run Distribution: Histogram



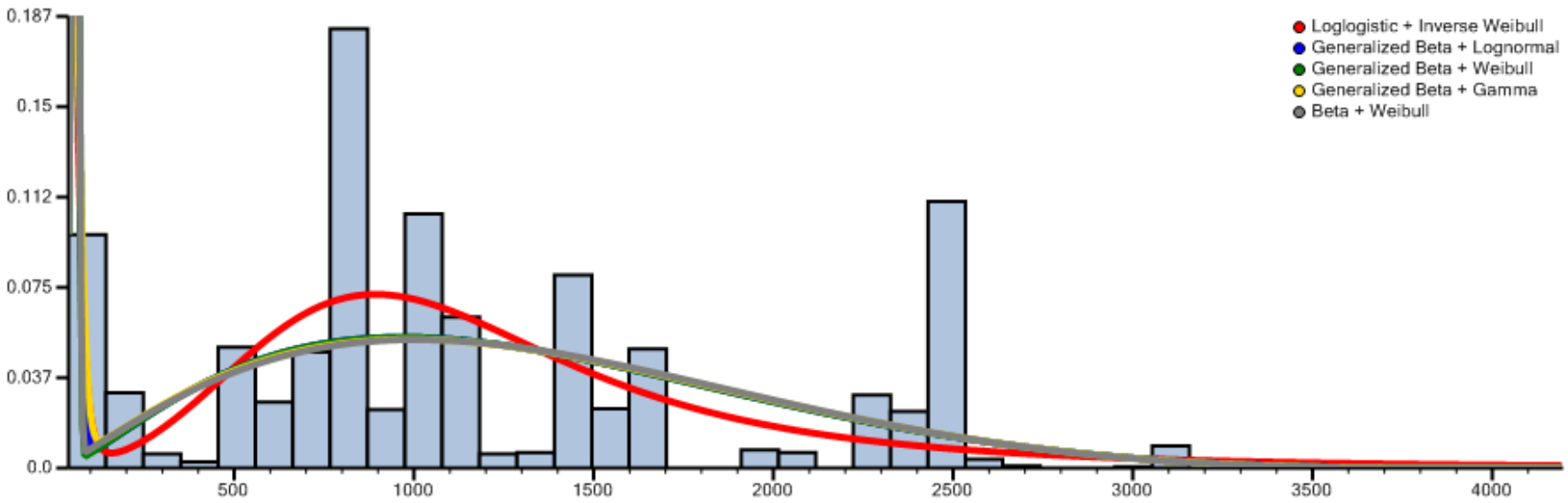
Parametric Model Analysis

- Obtained a sample of 100 **representative scenarios** from the full run distribution, using the **ModM2** method, to obtain the “sample run distribution” (Chueh and Johnson 2012, Johnson et al. 2013)
- The same data transformation that was applied to the full run distribution was then applied to the sample run distribution to obtain the “transformed sample run distribution” (65/100)
- We used AMOOF3 to accurately and efficiently fit 275 total parametric models to the transformed sample run distribution (single, mixed, and cross-mixed)
- We then determined goodness of fit by maximized loglikelihood value, and ranked the parametric models
- For the top 5 fitted parametric models, we compared CTE values at various levels to those of the transformed full run distribution

Top 5 Parametric Models: Ranked by Loglikelihood

Parametric Model	Loglikelihood
90.43% Generalized Beta(α = 10.98, β = 4.65, τ = 0.28, θ = 4201.42) + 9.57% Weibull(τ = 6.88, θ = 62.75)	-278,540
90.49% Beta(α = 2.27, β = 5.04, θ = 4205.71) + 9.51% Weibull(τ = 6.90, θ = 62.76)	-278,683
94.58% Loglogistic(γ = 2.38, θ = 1042.25) + 5.42% Inverse Weibull(τ = 275.55, θ = 67.42)	-278,879
90.44% Generalized Beta(α = 6.92, β = 4.71, τ = 0.41, θ = 4209.25) + 9.56% Gamma(α = 21.36, θ = 2.73)	-278,896
90.44% Generalized Beta(α = 14.47, β = 4.64, τ = 0.21, θ = 4205.44) + 9.56% Lognormal(μ = 4.04, σ = 0.21)	-278,932

Top 5 Parametric Models: Histograms



Top 5 Parametric Models: CTE Comparison to Trans Full Run

Parametric Model	Model CTE	(Model CTE/T. Full Run CTE)*100
90.43% Generalized Beta	CTE70 = 2087.48	104.73
+ 9.57% Weibull	CTE90 = 2583.86	106.74
	CTE99 = 3247.16	103.99
90.49% Beta	CTE70 = 2089.24	104.82
+ 9.51% Weibull	CTE90 = 2574.50	106.36
	CTE99 = 3211.11	103.15
94.58% Loglogistic	CTE70 = 2810.12	140.99
+ 5.42% Inverse Weibull	CTE90 = 4619.34	190.83
	CTE99 = 12,307.37	394.13
90.44% Generalized Beta	CTE70 = 2084.03	104.56
+ 9.56% Gamma	CTE90 = 2578.38	106.52
	CTE99 = 3240.33	103.77
90.44% Generalized Beta	CTE70 = 2113.85	107.72
+ 9.56% Lognormal	CTE90 = 2663.57	107.15
	CTE99 = 3142.91	100.65

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