# Actuarial Model Outcome Optimal Fit AMOOF 3.0

Presented to ARC 2014

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# Introduction

- Modeling single and mixed Probability Density Functions (pdfs)
- Integrating tail VaR and TVaR of the found model curves
- Optimization of the log-likelihood function
- Interactive and Graphical user interface
- Correcting small-sample bias correction for the maximum likelihood estimations

# Introduction ~cont'd

- Version 3.0 adds cross-mixed pdfs (22 times 22 pdfs) to allow mixing different density families, increases the speed of improper integral calculus, adds a real-time check on numeric accuracy, and includes a public website for the user to download the program and report testing issues for future improvement.
- Version 2.0 was sponsored by the Actuarial Foundation
  THE ACTUARIAL FOUNDATION

# **Projects Using AMOOF 3**

Stochastic Modeling Efficiency

 Loss Models Excel Tools: Simulator, Fitter, and Tester Tools

Real World Projects

# Analyzing the Seahawks Offensive **Play-Calling During** The 2013 Regular Season

Adam Brand May 2014

# MODELLING LION'S ROCK



Ben Squire Micah Darnall Paul Carpenter Christian Chmielewski

### AMOOF 3.0 Demonstration

James Smigaj Central Washington University



#### **ACCESS DATA PANEL**

#### Data Format

Pareto and Inver	rse Burr 2.csv - Notepad	
<u>F</u> ile <u>E</u> dit F <u>o</u> rmat	<u>V</u> iew <u>H</u> elp	
Pareto: Inverse Burr: VALUE	alpha = 7.62, theta = 91.67 tau = 4.83, theta = 12.4, gamma = 4.42 FREQUENCY	▲ H
0.36700945 6.178032596 32.37488264 10.61799004	, 1 , 1 , 1	
0.086463092 12.73476707 23.38107199 4.035153153		
20.51731523 16.03833077 3.318978924 0.236615459	, 1 , 1 , 1 , 1	
2.266575037 46.41125515 32.49606997 4.224946085	, 1 , 1 , 1 , 1	
13.67988768 4.04748201 22.83499422 52.95750367	, 1 , 1 , 1 , 1	
1.577043272 6.061290395 0.749309756	, 1 , 1 , 1 , 1	
11.02251442 4.061777904 44.71008188	, 1 , 1 , 1 , 1	
31.36918201	, 1 , 1	-

### Load Dataset



### Histogram bars



### Histogram range



### View dataset statistics





#### **MODEL SELECTION PANEL**

# Select pdfs

Access DataModel SelectionOptimize ModelsOrder ResultsFindingsUser Help1. Select PDFsPDF 1BetaPDF 2BurrIn ExponentialGammaGammaWork Qr Generalized BetaBeraBeraGeneralized ParetoBrowerse BurrInverse ExponentialInverse BurrInverse BurrInverse BurrInverse GaussianInverse GaussianInverse ParetoInverse ParetoInverse ParetoInverse VeibullLoglogisticParetoSingle Parameter ParetoTransformed GammaParetoSingle Parameter ParetoTransformed GammaWeibullWeibullWeibullMemovInverse Transformed GammaParetoSingle Parameter ParetoTransformed GammaWeibullTensformed GammaWeibullWeibullTensformed GammaWeibullTensformed GammaTensformed GammaTe	MOOF 3.0			
2. Set Initial ParametersPDF 1GammaBeta BurrBurrIm Exponential Generalized Pareto Burr $f(x) = \frac{(x,\theta')^2 e^{-x/\theta}}{xT(\alpha)}$ $E(x^4) = \theta^4 \Gamma(\alpha + k - 1) \cdots \alpha$ , if k is a positive integerWork Ga Beta Burr Comma Inverse Gamma Inverse Gamma Inverse Gamma Inverse Paratogistic Inverse Paratogistic Inverse Paratogistic Paratogistic Paratogistic Paratogistic Paratogistic Paratogistic ParatowMemoryImage: Colspan="2">CommaConstruct Paratogistic Paratogistic ParatowMemoryImage: Colspan="2">Colspan="2">Construct Paratogistic Paratogistic ParatowPerforma Inverse Paratogistic Paratogistic Paratogistic ParatowColspan="2">Construct Paratogistic Paratogistic ParatowPerforma Inverse Paratogistic Paratogistic Paratogistic ParatowColspan="2">Construct Paratogistic Paratogistic ParatowPerforma Paratogistic ParatowParatogistic ParatowConstruct Paratogistic ParatowPerforma Paratogistic ParatowParatogistic ParatowParatogistic ParatowPerforma Paratogistic ParatowParatogistic ParatowParatogistic ParatowPerforma ParatowParatogistic ParatowParatogistic ParatowPerforma ParatowParatogistic ParatowParatogistic ParatowPerforma ParatowParatogistic ParatowParatogistic ParatowPerforma ParatowParatogistic ParatowParatogistic ParatowPerforma ParatowParatogistic Paratow	Access Data Model Selection	Optimize Models Order Results	Findings	User Help
	1. Select PDFs PDF 1 Gamma Beta PDF 2 Burr Exponential Gamma Work Qu Generalized Beta Beta Burr Exponential Inverse Burr Camma Inverse Burr Camma Inverse Gamma Inverse Gaussian Inverse Gaussian Inverse Pareto Inverse Pareto Inverse Transformed Gamma Inverse Weibull Loglogistic Lognormal Paralogistic Pareto Single Parameter Pareto Transformed Beta Transformed Gamma Meibull Remove	2. Set Initial Parameters Gamma - $\alpha, \theta$ $f(x) = \frac{(x/\theta)^{\alpha}e^{-x/\theta}}{x\Gamma(\alpha)}$ $E[X^k] = \frac{\theta^k\Gamma(\alpha+k)}{\Gamma(\alpha)},  k > -\alpha$ $E[X^k] = \theta^k\Gamma(\alpha+k-1)\cdots\alpha, \text{ if k is a positive integer}$ 0.062 0.05 0.025 0.012 0.025 0.025 0.012 0.025 0.012 0.025 0.012 0.025 0.012 0.025 0.02	80 100	• Gamma

# Modify initial parameters



### **Parameter conditions**



### **Parameter conditions**



### Select starting parameters



# Mixed pdfs



# Mixed pdfs





#### **OPTIMIZE MODELS PANEL**

# **Run Solver**

Real Amoof 3.0				Iterations:	4072 Likelihood:	-7519.86538361521
				Iterations:	4073 Likelihood:	-7519.8653879108
				Iterations:	4074 Likelihood:	-7519.86654342334
Access Data	Model Selection Optin	nize Models	Order Results	Iterations:	4075 Likelihood:	-7519.86537951822
		0.) <i>(</i> ' 0.1 5		Iterations:	4076 Likelihood:	-7519.86551735152
1. Run Solver		3. View Solver F	Results	Iterations:	4077 Likelihood:	-7519.86537901951
Load Default 1.0	Initial Step Size	Moments		Iterations:	4078 Likelihood:	-7519.86562484621
Parameters		-1st: 0.194657	032702071	Iterations:	4079 Likelihood:	-7519.8653781972
Dup Cohor	Final Step Size	-2nd: Does not	exist.	Iterations:	4080 Likelihood:	-7519.86537780936
Rull Solver 5000	0 Iterations	-3rd: Does not	exist.	Iterations:	4081 Likelihood:	-7519.86537758034
Mark for Rerun		-4th: Does not	exist.	Iterations:	4082 Likelihood:	-7519.86537765968
		Mean Squared Fr	ror	Iterations:	4083 Likelihood:	-7519.86547769988
2. Review PDFs Ran		50805 403100502	2	Iterations:	4084 Likelihood:	-7519.86537721802
		03030.430130032	2	Iterations:	4085 Likelihood:	-7519.86537680725
Beta		LOCK Beta		Iterations:	4086 Likelihood:	-7519.86537605237
		ALPHA:	1.4044140484	Iterations:	4087 Likelihood:	- 1519.86531449139
Bui			45 440000700	Iterations:	4088 Likelinood:	-1519.86531148851
Exponential		BETA:	15.118060766	Iterations:	4089 Likelinood:	- 1519.86536530621
			107 1808032	Iterations:	4090 Likelinood:	-7519.86535355109
Gamma			137.10000320	Iterations:	4091 Likelihood:	-7519.86533001321
Parata + Invarca Rurr				Iterations:	4052 Likelihood:	-7519.06520619463
				Iterations:	4093 Likelihood:	-7519.86518543361
				Iterations:	4054 Likelihood:	-7519.86500133145
				Iterations:	4095 Likelihood:	-7519 00207059/02
		0.062 <b>]</b>		Iterations:	4096 Likelihood:	-7519 26227922612
		0.05		Iterations:	4098 Likelihood.	-7519 25965614222
		0.05		Iterations:	4099 Likelihood.	-7519 25422497512
		0.037 -		Iterations:	4100 Likelihood.	-7519 8475403277
			տ	Iterations:	4101 Likelihood.	-7540 82564445384
		0.025	Ulh I	Iterations:	4102 Likelihood	-7519 84271541799
				Iterations:	4103 Likelihood	-7519 83953047281
		0.012 -		Iterations:	4104 Likelihood	-7519 83543198361
				Iterations:	4105 Likelihood	-7519,83399511982
		5.0	20 40	Iterations:	4106 Likelihood	-7519.84232041647
	F F	Į		Iterations:	4107 Likelihood:	-28682.0603838575
	Running Solver: Optimizi	ng Burr Overall Co	mpletion: 20.000	Iterations:	4108 Likelihood:	-14265.9884044567
	Running Solver. Optimizi	ng bun overall co	mpletion. 20.007	Iterations	4109 Likelihood	-7610 0238991635

# **BOBYQA** solver

- Bounded Optimization by Quadratic Approximation.
- Released 2009.
- C# port of FORTRAN library
- Uses interpolation points to approximate and maximize the objective function at each iteration.











# **Resulting Log-Likelihood**

PDF	<b>Before Optimization</b>	After Optimization
Beta	-7576.85	-7542.31
Burr	-27243.86	-7519.82
Exponential	-7965.81	-7643.14
Gamma	-10164.99	-7553.65
Pareto + Inverse Burr	-7371.41	-7312.24

### **Comparison with Expected Parameters**



### **Comparison with Expected Parameters**

#### Pareto

Parameter	Expected Value	Found Value	% Difference
alpha	7.62	13.78	80.85
theta	91.67	170.78	86.30
weight	0.50	0.53	6.84

#### **Inverse Burr**

Parameter	Expected value	Found value	% Difference
gamma	4.42	4.38	-1.00
tau	4.83	53.38	1005.20
theta	12.40	6.99	-43.59
weight	0.50	0.47	-6.84

However, the fitted function attains a better loglikelihood than the expected function.

- expected : -7316.92
- found : -7312.24

HAMOOF 3.0	D												. 🗆 🗙	
Acce	ess Data	Mo	del Select	tion	Optimize	Models	Orde	r Results		Findings		User H	lelp	
1. Choos	e PDFs to	o Integrate	e				2. View	Moments	and Integ	rate				
PDF Na	me				Average Er	ror	Calcu	late VARs	/CTEs	Calcula	te All			
Gamma	1				28248.2	28248.257386 Sample Moments Model Moments %						6 Difference		
Beta					59895.6	95740	-1st: -2nd:	0.29956049 4.00123679	199 -1st 527 -2nd	Does not Does not	t exist1 t exist2	lst: NA nd: NA		
Burr					95973.2	98408	-3rd:	172.550460	742 -3rd	Does not	t exist3	rd: NA		
Pareto +	⊦ Inverse Burr				109970	.06501	-4th: 1st: 2nd:	8913.09041 16.8032222 432 937043	792 -4th: 512 1st 725 2pd	Does not 16.80193 434.0459	t exist4 336450 1 904255 2	hth: NA 1st: -0.007 nd: 0.2561	66880394	
Exponen	iuai				524156	43039	3rd: 4th:	16028.2800 865050.396	443 3rd 243 4th	16433.47 1084990	716847 3 .15302 4	rd: 2.5279 http://discourse.org/10.2007 http://discourse.org/10.2007	079541826 508017338	
-3. Comp	are Datas	et and Mo	del VARs	/CTEs		,							,	
	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.161450	5.942884	8.028761	10.533326	12.839634	14.185402	15.210637	16.137382	
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556	
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.023382	-1.120152	1.625640	-0.399091	-0.433336	-0.117728	0.334926	-0.025861	
L CTE PDF	0.117929	0.237445	0.358593	0.605991	1.256867	1.960693	2.728133	3.573771	4.518370	5.556562	6.556714	7.462831	8.284329	
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257	
% Diff	-15.349090	-7.520448	-1.669328	0.321322	-3.358050	-3.520356	-2.473375	-1.099480	-0.474785	-0.300363	-0.180994	-0.248641	-0.155802	
	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%	
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.649888	23.451184	25.930174	29.806796	37.593895	44.212447	49.913103	60.419070	
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397	
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.097160	-0.095861	-0.434159	-0.719938	2.114271	2.943534	2.147532	-3.378026	
R CTE PDF	25.319535	26.289173	27.385083	28.652355	30.157437	32.005758	34.379764	37.634810	42.603342	52.073756	59.745520	66.197984	77.905524	
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116	
% Diff	0.004622	-0.024351	-0.061253	-0.101314	-0.121019	-0.039279	0.053229	0.080550	0.284499	-0.034003	-0.885958	-2.029665	-2.446298	

#### **ORDER RESULTS PANEL**

### **Compare Moments**

AMOOF 3.0													. 🗆 🗙	
Acce	ss Data	Mo	del Select	ion	Optimize I	Models	Order	Results		Findings		User H	lelp	
1. Choos	e PDFs to	Integrate	÷				2. View	Moments	and Integr	ate				
PDF Na	me				Average Er	ror	Calcul	ate VARs	CTEs	Calculat	e All			
🔵 Gamma					28248.2	57386	Sample I	Noments	Mode	el Moments	%	% Difference		
Beta					59895 6	95740	-1st: (	0.299560491	99 -1st:	Does not	exist1	st: NA		
					00000.0		-2nd: 4	1.001236795	527 -2nd:	Does not	exist2	nd: NA		
Burr					95973.2	98408	-3rd: 1	72.5504607	42 -3rd:	Does not	exist3	rd: NA		
Pareto +	Inverse Burr				109970	06501	-4th: 8	3913.090417	'92 -4th:	Does not	exist4	th: NA	000000	
							1st: 1	16.80322225	512 1st:	16.80193	36450 1	st: -0.007	66880394	
Exponen	ntial				524158.	43859	2nd: 4	132.9370437	25 2nd:	434.0459	04255 21	nd: 0.256	125121804	
							3rd: 1	16028.28004	43 3rd:	16433.47	16847 3	rd: 2.5279	979541826	
							4th: 8	365050.3962	243 4th:	1084990.	15302 4	th: 25.42	08017338	
3. Compa	are Datas	et and Mo	del VARs	/CTEs										
	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.161450	5.942884	8.028761	10.533326	12.839634	14.185402	15.210637	16.137382	
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556	
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.023382	-1.120152	1.625640	-0.399091	-0.433336	-0.117728	0.334926	-0.025861	
L CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257	
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%	
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21.649888	23.451184	25.930174	29.806796	37.593895	44.212447	49.913103	60.419070	
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397	
% Diff	-0.025861	0.707099	0.135724	0.386957	-0.446851	-1.097160	-0.095861	-0.434159	-0.719938	2.114271	2.943534	2.147532	-3.378026	
R CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116	
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
TEs not ca	alculated.													

### **Compare Moments**

AMOOF 3.0	)												. 🗆 🗙	
Acce	ess Data	Mo	del Select	ion (	Optimize I	Models	Order	Results		Findings		User H	lelp	
1. Choos	e PDFs to	Integrate	; 				2. View I	Moments a	and Integr	ate				
PDF Na	me				Average Err	or	Calcul	Iate VARs/CTEs Calculate All						
Gamma					28248.2	57386	Sample Moments Model Moments					% Difference		
Beta					59895.6	95740	-1st: 0	).299560491	99 -1st:	Does not	exist1	st: NA		
							-2nd: 4	001236795	27 -2nd:	Does not	exist21	nd: NA		
Burr					95973.2	98408	-3rd: 1	012.5504607	42 -3rd:	Does not	exist3	ra: NA		
Pareto +	Inverse Burr				109970.	06501	-4th: 0	913.090417 6 80322225	92 -4th:	16 80322	exist4	In NA	06	
Exponer					524158	43859	2nd: 4	32.9370437	25 2nd	564.6965	96574 2	nd: 30.433	388288409	
					024100.	40000	3rd: 1	6028.28004	43 3rd:	28466.16	82717 3	rd: 77.599	964383529	
							4th: 8	365050.3962	.43 4th:	1913293.	47709 4	th: 121.17	771112302	
2 Comp	ara Dataa	at and Ma	del \/A De				[							
S. Compa	are Datas	et and ivid		CIES										
	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993289	7.238541	8.583517	10.045588	11.647107	
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556	
% Diff	-40.360426	-27.489272	-28.513502	-31.709278	-35.030534	-36.361648	-37.613954	-38.812858	-43.328726	-43.867760	-39.561731	-33.735624	-27.843967	
L CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257	
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%	
VAR PDF	11.647107	13.417503	15.396637	17.640395	20.230623	23.294213	27.043744	31.877730	38.690850	50.337957	58.921474	65.734594	77.381702	
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397	
% Diff	-27.843967	-20.752080	-14.363685	-7.010863	-0.445408	6.414583	15.208766	22.403071	28.870948	36.730281	37.191791	34.526329	23.748557	
R CTE PDF	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116	
% Diff	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
				·	·				·					
CTEs not ca	alculated.													

### Calculate CTEs

	n						
	•					-	<pre>*** allocatemem: warning: doubling stack size; new stack = 64000000 ( realprecision = 105 significant digits (100 digits displayed)</pre>
Acce	ess Data	Mo	del Selec	tion	Optimize	Models	·····
-1 Choos	se PDEs to	o Integrate	e				entering calculateAllVARCTEs
PDF Na	ame	, intograti	•		Average Er	ror	Function Component 1: 0.538371431825554×Pareto_(xVar,16.2034719757771,204.395074453393)
Gamma	3				28248.2	257386	Function Commonant 2.
Beta					59895.6	695740	0.461628568174446×InverseBurr_(xVar,4.40223310128007,50.3932108251082,7
Burr					95973 2	298408	PDF Subcomponent Areas:
Pareto -					109970	.06501	Expected: 0.538371431 Found: 0.538371430 Error: 0.00000000260362495 Expected: 0.461628568 Found: 0.461628567 Error: 0.00000000234239567
Expone	ntial				524158	.43859	· TOTAL AREA: 0.9999999975169659 Bounds:
	<b>.</b>						
3. Comp	are Datas	et and Mo		S/CIES			calculate VARCTE. Expected Area: 0.010000000000 Found Area: 0.0100000000
	1%	2%	3%	5%	10%	15%	calculate VARCTE. Expected Area: 0.02000000000 Found Area: 0.0200000000
VAR PDF	0.236645	0.478093	0.724538	1.233250	2.608513	4.16145	calculate VARCTE. Expected Area: 0.03000000000 Found Area: 0.0300000000
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.29118	calculate VARCTE. Expected Area: 0.05000000000 Found Area: 0.0500000000
% Diff	-16.428533	2.120374	1.198481	-2.285338	-4.273563	-3.0233	calculate VARCTE. Expected Area: 0.1000000000 Found Area: 0.1000000000
L CTE PDF	0.117929	0.237445	0.358593	0.605991	1.256867	1.96069	calculate VARCTE. Expected Area: 0.1500000000 Found Area: 0.1500000000
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.03223	calculate VARCTE. Expected Area: 0.2000000000 Found Area: 0.2000000000
% Diff	-15.349090	-7.520448	-1.669328	0.321322	-3.358050	-3.5203	calculate VARCTE. Expected Area: 0.2500000000 Found Area: 0.2500000000
	50%	55%	60%	65%	70%	75%	calculate VARCTE. Expected Area: 0.3000000000 Found Area: 0.3000000000
VAR PDF	16.137382	17.050767	18.003500	19.043790	20.230330	21,6498	calculate VARCTE. Expected Area: 0.3500000000 Found Area: 0.3500000000
VAR Data	16 141556	16 931047	17 979098	18 970383	20 321135	21 8900	calculate VARCTE. Expected Area: 0.4000000000 Found Area: 0.4000000000
% Diff	-0.025861	0 707099	0 135724	0.386957	-0 446851	-1 0971	calculate VARCTE. Expected Area: 0.4500000000 Found Area: 0.4500000000
R CTE PDF	25.319535	26,289173	27.385083	28.652355	30,157437	32.0057	calculate VARCTE. Expected Area: 0.5000000000 Found Area: 0.5000000000
R CTF Data	25 318365	26 295577	27 401867	28 681414	30 193978	32 0183	calculate VARCTE. Expected Area: 0.5500000000 Found Area: 0.5500000000
% Diff	0.004622	-0.024351	-0.061253	-0 101314	-0 121019	-0.0392	calculate VARCTE. Expected Area: 0.6000000000 Found Area: 0.6000000000
					3.121010		calculate VARCTE. Expected Area: 0.6500000000 Found Area: 0.6500000000
							calculate VARCTE. Expected Area: 0.7000000000 Found Area: 0.7000000000

# Compare VaRs/CTEs

Acce	ss Data	Мо	del Select	ion	Optimize I	Models	Orde	r Results		Findings		User H	lelp	
I. Choos	e PDFs to	o Integrate	e				-2. View	Moments	and Integ	ate				
PDF Na	me				Average Err	or	Calculate VARs/CTEs Calculate All							
Gamma					28248.2	57386	Sample	Moments	Mode	el Moments	%	% Difference		
Beta					59895.695740 -1st: 0.2995604 -2nd: 4.0012367				049199 -1st: Does not exist.			lst: NA nd: NA		
Burr					95973.2	98408	-3rd:	172.5504607	7 <b>42 -3rd</b> :	Does not	exist3	rd: NA		
Pareto + Inverse Burr					109970.	06501	-4th: 1st:	8913.090417 16.80322225	792 -4th: 512 1st:	Does not 16.80193	exist4 336450 1	th: NA st: -0.007	6688039	
Exponential					524158.43859      2nd:      432.937043725      2nd:      434      3rd:      16028.2800443      3rd:      164      4th:      865050.396243      4th:      108<			434.0459 16433.47 1084990	5904255      2nd:      0.256125121804        4716847      3rd:      2.527979541826        00.15302      4th:      25.42508017338					
3 Compare Dataset and Model VARs/CTEs							iui.							
3. Compa	are Datas	et and Mo	odel VARs	/CTEs		]								
3. Compa	are Datas	et and Mo	odel VARs	/CTEs	10%	15%	20%	25%	30%	35%	40%	45%	50%	
3. Compa	are Datas 1% 0.236645	et and Mo 2% 0.478093	odel VARs. 3% 0.724538	/CTEs 5% 1.233250	10% 2.608513	15% 4.161450	20% 5.942884	25% 8.028761	30% 10.533326	35% 12.839634	40% 14.185402	45% 15.210637	50% 16.13738	
3. Compa /AR PDF /AR Data	are Datas 1% 0.236645 0.283164	et and Mo 2% 0.478093 0.468166	0del VARs 3% 0.724538 0.715957	/CTEs 5% 1.233250 1.262093	10% 2.608513 2.724967	15% 4.161450 4.291189	20% 5.942884 6.010207	25% 8.028761 7.900330	30% 10.533326 10.575532	35% 12.839634 12.895515	40% 14.185402 14.202122	45% 15.210637 15.159863	50% 16.13738 16.14155	
AR PDF AR Data Off	are Datas 1% 0.236645 0.283164 -16.428533	et and Mo 2% 0.478093 0.468166 2.120374	odel VARs 3% 0.724538 0.715957 1.198481	/CTEs 5% 1.233250 1.262093 -2.285338	10% 2.608513 2.724967 -4.273563	15% 4.161450 4.291189 -3.023382	20% 5.942884 6.010207 -1.120152	25% 8.028761 7.900330 1.625640	30% 10.533326 10.575532 -0.399091	35% 12.839634 12.895515 -0.433336	40% 14.185402 14.202122 -0.117728	45% 15.210637 15.159863 0.334926	50% 16.13738 16.14155 -0.02586	
3. Compa /AR PDF /AR Data % Diff _CTE PDF	are Datas 1% 0.236645 0.283164 -16.428533 0.117929	et and Mo 2% 0.478093 0.468166 2.120374 0.237445	3% 0.724538 0.715957 1.198481 0.358593	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991	10% 2.608513 2.724967 -4.273563 1.256867	15% 4.161450 4.291189 -3.023382 1.960693	20% 5.942884 6.010207 -1.120152 2.728133	25% 8.028761 7.900330 1.625640 3.573771	30% 10.533326 10.575532 -0.399091 4.518370	35% 12.839634 12.895515 -0.433336 5.556562	40% 14.185402 14.202122 -0.117728 6.556714	45% 15.210637 15.159863 0.334926 7.462831	50% 16.13738 16.14155 -0.02586 8.284329	
3. Compa /AR PDF /AR Data % Diff _ CTE PDF _ CTE Data	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312	et and Mc 2% 0.478093 0.468166 2.120374 0.237445 0.256754	3% 0.724538 0.715957 1.198481 0.358593 0.364681	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050	10% 2.608513 2.724967 -4.273563 1.256867 1.300539	15% 4.161450 4.291189 -3.023382 1.960693 2.032235	20% 5.942884 6.010207 -1.120152 2.728133 2.797321	25% 8.028761 7.900330 1.625640 3.573771 3.613501	30% 10.533326 10.575532 -0.399091 4.518370 4.539925	35% 12.839634 12.895515 -0.433336 5.556562 5.573302	40% 14.185402 14.202122 -0.117728 6.556714 6.568603	45% 15.210637 15.159863 0.334926 7.462831 7.481432	50% 16.13738 16.14155 -0.02586 8.284329 8.297257	
3. Compa /AR PDF /AR Data /6 Diff _ CTE PDF _ CTE Data /6 Diff	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448	odel VARs        3%        0.724538        0.715957        1.198481        0.358593        0.364681        -1.669328	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.15580	
3. Compa /AR PDF /AR Data % Diff _ CTE PDF _ CTE Data % Diff	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50%	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55%	3%        0.724538        0.715957        1.198481        0.358593        0.364681        -1.669328        60%	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65%	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70%	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75%	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80%	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85%	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90%	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95%	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97%	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98%	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.15580 99%	
3. Compa /AR PDF /AR Data % Diff . CTE PDF . CTE Data % Diff /AR PDF	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50% 16.137382	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55% 17.050767	3%        0.724538        0.715957        1.198481        0.358593        0.364681        -1.669328        60%        18.003500	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65% 19.043790	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70% 20.230330	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75% 21.649888	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80% 23.451184	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85% 25.930174	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90% 29.806796	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95% 37.593895	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97% 44.212447	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98% 49.913103	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.15580 99% 60.41907	
3. Compa /AR PDF /AR Data % Diff _CTE PDF _CTE Data % Diff /AR PDF /AR Data	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50% 16.137382 16.141556	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55% 17.050767 16.931047	Odel VARs        3%        0.724538        0.715957        1.198481        0.358593        0.364681        -1.669328        60%        18.003500        17.979098	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65% 19.043790 18.970383	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70% 20.230330 20.321135	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75% 21.649888 21.890057	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80% 23.451184 23.473686	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85% 25.930174 26.043243	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90% 29.806796 30.022942	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95% 37.593895 36.815515	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97% 44.212447 42.948250	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98% 49.913103 48.863739	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.155802 99% 60.41907 62.53139	
AR PDF /AR Data % Diff _ CTE PDF _ CTE Data % Diff /AR PDF /AR Data % Diff	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50% 16.137382 16.141556 -0.025861	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55% 17.050767 16.931047 0.707099	Odel VARs        3%        0.724538        0.715957        1.198481        0.358593        0.364681        -1.669328        60%        18.003500        17.979098        0.135724	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65% 19.043790 18.970383 0.386957	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70% 20.230330 20.321135 -0.446851	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75% 21.649888 21.890057 -1.097160	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80% 23.451184 23.473686 -0.095861	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85% 25.930174 26.043243 -0.434159	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90% 29.806796 30.022942 -0.719938	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95% 37.593895 36.815515 2.114271	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97% 44.212447 42.948250 2.943534	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98% 49.913103 48.863739 2.147532	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.155802 99% 60.41907 62.53139 -3.378020	
AR PDF AR Data % Diff CTE PDF CTE Data % Diff AR PDF AR Data % Diff R CTE PDF	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50% 16.137382 16.141556 -0.025861 25.319535	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55% 17.050767 16.931047 0.707099 26.289173	Odel      VARs        3%      0.724538        0.715957      1.198481        0.358593      0.364681        -1.669328      60%        18.003500      17.979098        0.135724      27.385083	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65% 19.043790 18.970383 0.386957 28.652355	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70% 20.230330 20.321135 -0.446851 30.157437	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75% 21.649888 21.890057 -1.097160 32.005758	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80% 23.451184 23.473686 -0.095861 34.379764	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85% 25.930174 26.043243 -0.434159 37.634810	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90% 29.806796 30.022942 -0.719938 42.603342	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95% 37.593895 36.815515 2.114271 52.073756	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97% 44.212447 42.948250 2.943534 59.745520	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98% 49.913103 48.863739 2.147532 66.197984	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.155802 99% 60.41907 62.53139 -3.378022 77.90552	
AR PDF /AR Data % Diff CTE PDF CTE Data % Diff /AR PDF /AR Data % Diff R CTE PDF R CTE PDF R CTE Data	are Datas 1% 0.236645 0.283164 -16.428533 0.117929 0.139312 -15.349090 50% 16.137382 16.141556 -0.025861 25.319535 25.318365	et and Mo 2% 0.478093 0.468166 2.120374 0.237445 0.256754 -7.520448 55% 17.050767 16.931047 0.707099 26.289173 26.295577	Odel      VARs        3%      0.724538        0.715957      1.198481        0.358593      0.364681        -1.669328      60%        18.003500      17.979098        0.135724      27.385083        27.401867      10.1867	/CTEs 5% 1.233250 1.262093 -2.285338 0.605991 0.604050 0.321322 65% 19.043790 18.970383 0.386957 28.652355 28.681414	10% 2.608513 2.724967 -4.273563 1.256867 1.300539 -3.358050 70% 20.230330 20.321135 -0.446851 30.157437 30.193978	15% 4.161450 4.291189 -3.023382 1.960693 2.032235 -3.520356 75% 21.649888 21.890057 -1.097160 32.005758 32.018334	20% 5.942884 6.010207 -1.120152 2.728133 2.797321 -2.473375 80% 23.451184 23.473686 -0.095861 34.379764 34.361474	25% 8.028761 7.900330 1.625640 3.573771 3.613501 -1.099480 85% 25.930174 26.043243 -0.434159 37.634810 37.604520	30% 10.533326 10.575532 -0.399091 4.518370 4.539925 -0.474785 90% 29.806796 30.022942 -0.719938 42.603342 42.482480	35% 12.839634 12.895515 -0.433336 5.556562 5.573302 -0.300363 95% 37.593895 36.815515 2.114271 52.073756 52.091469	40% 14.185402 14.202122 -0.117728 6.556714 6.568603 -0.180994 97% 44.212447 42.948250 2.943534 59.745520 60.279572	45% 15.210637 15.159863 0.334926 7.462831 7.481432 -0.248641 98% 49.913103 48.863739 2.147532 66.197984 67.569416	50% 16.13738 16.14155 -0.02586 8.284329 8.297257 -0.15580 99% 60.41907 62.53139 -3.37802 77.90552 79.85911	

# Compare VaRs/CTEs

AMOOF 3.0											_		. <b>D</b> X	
Acce	ss Data	Mo	del Select	ion	Optimize I	Models	Orde	r Results	T	Findings		User H	lelp	
1. Choos	e PDFs to	Integrate	; 				2. View	Moments	and Integr	ate				
PDF Na	me				Average Err	or	Calculate VARs/CTEs Calculate All							
🔵 Gamma					28248.257386 Sample Moments Model Moments						%	% Difference		
Beta					59895.6	95740	-1st: ( -2nd: 4	0.299560491 4.001236795	99 -1st: 527 -2nd:	Does not Does not	exist1 exist2r	st: NA nd: NA		
📄 Burr					95973.2	98408	-3rd: 1	172.5504607	'42 -3rd:	Does not	exist3	rd: NA		
Pareto +	Inverse Burr				109970.	06501	-4th: { 1st: 1	3913.090417 16.80322225 432 9370437	792 -4th: 512 1st: 725 2nd:	Does not 16.80322 564.6965	exist4 28541 1 96574 2r	th: NA st: 3.58e- nd: 30.433	06	
					524156.	43039	3rd: 4th: 8	16028.28004 365050.3962	23 21d. 143 3rd: 243 4th:	28466.16 1913293.	82717 3 47709 4	rd: 77.599 th: 121.17	964383529 771112302	
3. Compa	are Datas	et and Mo	del VARs	/CTEs										
	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993289	7.238541	8.583517	10.045588	11.647107	
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556	
% Diff	-40.360426	-27.489272	-28.513502	-31.709278	-35.030534	-36.361648	-37.613954	-38.812858	-43.328726	-43.867760	-39.561731	-33.735624	-27.843967	
L CTE PDF	0.084298	0.169164	0.254607	0.427262	0.869657	1.328453	1.805100	2.301265	2.818883	3.360217	3.927948	4.525282	5.156116	
L CTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257	
% Diff	-39.490285	-34.114436	-30.183597	-29.267074	-33.131063	-34.630951	-35.470416	-36.314814	-37.909039	-39.708680	-40.201168	-39.513167	-37.857578	
	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%	
VAR PDF	11.647107	13.417503	15.396637	17.640395	20.230623	23.294213	27.043744	31.877730	38.690850	50.337957	58.921474	65.734594	77.381702	
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397	
% Diff	-27.843967	-20.752080	-14.363685	-7.010863	-0.445408	6.414583	15.208766	22.403071	28.870948	36.730281	37.191791	34.526329	23.748557	
R CTE PDF	28.450329	30.220725	32.199859	34.443617	37.033845	40.097434	43.846965	48.680950	55.494070	67.141173	75.724685	82.537799	94.184883	
R CTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116	
% Diff	12.370326	14.927028	17.509727	20.090374	22.653084	25.232731	27.605018	29.455052	30.628132	28.890918	25.622465	22.152601	17.938799	



#### **FINDINGS PANEL**

#### **AMOOF Risk Analysis Report**

7/9/2014 3:01:35 PM

Report generated by AMOOF 3.0

Data File: Pareto and Inverse Burr 2.csv

Dataset	Statistics	Dataset Moments		
Mean	16.803222251257	-1st	0.299560491996276	
Median	16.14110986	-2nd	4.00123679527133	
Q1	7.900329808	-3rd	172.550460742789	
Q3	21.89005726	-4th	8913.09041792718	
Min	0.017323854	1st	16.803222251257	
Max	135.2766001	2nd	432.937043725345	
Std Dev	12.2306486214022	3rd	16028.2800443683	
Count	2000	4th	865050.396243577	



PDFs

Pareto + Inverse Burr Moment RMSE (existing moments only): 120071.887135944 Log-Likelihood: -7312.19947734458 Kolmogorov Smirnov Test: 0.00949980379491816 Anderson-Darling Test: 0.153311788554447



Moments						
Moment	Data	Model				
-1st	0.299560491996276	Does not exist.				
-2nd	4.00123679527133	Does not exist.				
-3rd	172.550460742789	Does not exist.				
-4th	8913.09041792718	Does not exist.				
1st	16.803222251257	16.8049208199086				
2nd	432.937043725345	434.721666975078				
3rd	16028.2800443683	16530.7870088455				
4th	865050.396243577	1105193.64475395				

Parameters					
ALPHA	15.4635167401687				
THETA	194.184881033639				
GAMMA2	4.39451791834518				
TAU2	53.2247780858357				
THETA2	7.0236804754546				
WEIGHT1	0.537401854797764				
WEIGHT2	0.462598145202236				

#### Conditional Tail Expectations and Value at Risks:

	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.236017	0.476848	0.722687	1.230222	2.602825	4.153703	5.934075	8.020584	10.528928	12.839738	14.185746	15.211064	16.137973
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-16.650010	1.854527	0.939890	-2.525274	-4.482330	-3.203915	-1.266721	1.522142	-0.440672	-0.432525	-0.115310	0.337741	-0.022197
LCTE PDF	0.117615	0.236819	0.357660	0.604454	1.253899	1.956454	2.722858	3.567816	4.512296	5.551140	6.552005	7.458687	8.280650
LCTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-15.574692	-7.764062	-1.925254	0.066768	-3.586220	-3.728932	-2.661949	-1.264285	-0.608565	-0.397651	-0.252686	-0.304029	-0.200144

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	16.137973	17.051582	18.004571	19.045122	20.231898	21.651619	23.452934	25.931758	29.808479	37.602189	44.236835	49.959690	60.525372
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-0.022197	0.711912	0.141678	0.393981	-0.439133	-1.089255	-0.088403	-0.428078	-0.714330	2.136800	3.000319	2.242872	-3.208028
RCTE PDF	25.329188	26.299822	27.396944	28.665740	30.172810	32.023874	34.401970	37.663858	42.646142	52.155781	59.872457	66.371393	78.181652
RCTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	0.042751	0.016143	-0.017965	-0.054647	-0.070105	0.017301	0.117853	0.157796	0.385245	0.123459	-0.675378	-1.773026	-2.100529

Exponential Moment RMSE (existing moments only): 524158.301767285 Log-Likelihood: -7643.14133737572 Kolmogorov Smirnov Test: 0.187578661893205 Anderson-Darling Test: 87.5504631251774



Moments						
Moment	Data	Model				
-1st	0.299560491996276	Does not exist.				
-2nd	4.00123679527133	Does not exist.				
-3rd	172.550460742789	Does not exist.				
-4th	8913.09041792718	Does not exist.				
1st	16.803222251257	16.8032222534041				
2nd	432.937043725345	564.696556194588				
3rd	16028.2800443683	28466.1652184086				
4th	865050.396243577	1913293.20346817				

Parameters					
THETA 16.8032222534041					

#### Conditional Tail Expectations and Value at Risks:

	1%	2%	3%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
VAR PDF	0.168878	0.339471	0.511813	0.861893	1.770396	2.730842	3.749531	4.833986	5.993288	7.238541	8.583517	10.045588	11.647106
VAR Data	0.283164	0.468166	0.715957	1.262093	2.724967	4.291189	6.010207	7.900330	10.575532	12.895515	14.202122	15.159863	16.141556
% Diff	-40.360426	-27.489272	-28.513502	-31.709282	-35.030536	-36.361650	-37.613956	-38.812861	-43.328728	-43.867762	-39.561733	-33.735627	-27.843970
LCTE PDF	0.084298	0.169164	0.254607	0.427262	0.869657	1.328453	1.805099	2.301265	2.818883	3.360217	3.927948	4.525281	5.156116
LCTE Data	0.139312	0.256754	0.364681	0.604050	1.300539	2.032235	2.797321	3.613501	4.539925	5.573302	6.568603	7.481432	8.297257
% Diff	-39.490283	-34.114434	-30.183594	-29.267080	-33.131065	-34.630953	-35.470419	-36.314816	-37.909042	-39.708682	-40.201170	-39.513169	-37.857581

	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	97%	98%	99%
VAR PDF	11.647106	13.417502	15.396637	17.640394	20.230623	23.294212	27.043743	31.877729	38.690849	50.337955	58.921472	65.734592	77.381698
VAR Data	16.141556	16.931047	17.979098	18.970383	20.321135	21.890057	23.473686	26.043243	30.022942	36.815515	42.948250	48.863739	62.531397
% Diff	-27.843970	-20.752083	-14.363688	-7.010867	-0.445411	6.414579	15.208762	22.403067	28.870944	36.730275	37.191786	34.526325	23.748551
RCTE PDF	28.450328	30.220724	32.199858	34.443616	37.033844	40.097433	43.846963	48.680949	55.494067	67.141171	75.724682	82.537795	94.184885
RCTE Data	25.318365	26.295577	27.401867	28.681414	30.193978	32.018334	34.361474	37.604520	42.482480	52.091469	60.279572	67.569416	79.859116
% Diff	12.370322	14.927023	17.509723	20.090370	22.653079	25.232727	27.605013	29.455047	30.628126	28.890915	25.622461	22.152595	17.938802

# Help and Downloads

• Website:

https://bitbucket.org/AMOOF3/amoof-3.0/wiki/Home

- Download:
- <u>https://bitbucket.org/AMOOF3/amoof-</u> <u>3.0/downloads</u>

#### AMOOF3: Stochastic Efficient Modeling Application

Yvonne C. Chueh, PhD, ASA Paul H. Johnson, Jr., PhD James Smigaj

Joint work between the University of Illinois at Urbana-Champaign (UIUC) and Central Washington University (CWU) Funded by The Actuarial Foundation





 Used AMOOF3 (Actuarial Model Optimal Outcome Fit V3.0) to analyze statutory ending surplus data from a real block of universal life insurance, provided by Milliman, i.e. "Milliman data"



- Present value of ending surplus data at 30 years (360 months), i.e, "ending surplus," was the output from a real block of universal life insurance using a proprietary stochastic scenario generator
- 50,000 stochastic economic 7-year US treasury yield scenarios were considered, where each scenario is a random path of monthly portfolio yield rates x = (r<sub>1</sub>, r<sub>2</sub>, ..., r<sub>360</sub>)
- We called the 50,000 ending surplus data the "full run distribution," (true distribution)

#### **Data Transformation**

• The full run distribution was transformed by:

- Dividing all ending surplus values by 1000
- Muliplying all ending surplus values by negative one
- Ignoring any negative values, corresponding to a positive ending surplus value
- Therefore, we only focused on the tail distribution for ending surplus (the worst ending surplus values, 38,137 of the original 50,000 observations)
- The resulting distribution was the "transformed full run distribution," and various statistics were accurately and efficiently computed using AMOOF3

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**Full Run Distribution Analysis** 

#### Trans Full Run Distribution: Statistics

Statistic Name	Statistic Value
Mean	1172.79
Median	1135.62
Standard Deviation	714.51
Minimum	0.14
Maximum	48,341.36
CTE70	1993.12
CTE90	2420.63
CTE99	3122.65

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**Full Run Distribution Analysis** 

#### Trans Full Run Distribution: Histogram





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#### Parametric Model Analysis

- Obtained a sample of 100 representative scenarios from the full run distribution, using the ModM2 method, to obtain the "sample run distribution" (Chueh and Johnson 2012, Johnson et al. 2013)
- The same data transformation that was applied to the full run distribution was then applied to the sample run distribution to obtain the "transformed sample run distribution" (65/100)
- We used AMOOF3 to accurately and efficiently fit 275 total parametric models to the transformed sample run distribution (single, mixed, and cross-mixed)
- We then determined goodness of fit by maximized loglikelihood value, and ranked the parametric models
- For the top 5 fitted parametric models, we compared CTE values at various levels to those of the transformed full run distribution

#### Top 5 Parametric Models: Ranked by Loglikelihood

Parametric Model	Loglikelihood
90.43% Generalized Beta( $\alpha$ = 10.98, $\beta$ = 4.65, $\tau$ = 0.28, $\theta$ = 4201.42)	-278,540
+ 9.57% Weibull( $\tau$ = 6.88, $\theta$ = 62.75)	
90.49% Beta( $\alpha$ = 2.27, $\beta$ = 5.04, $\theta$ = 4205.71)	-278,683
+ 9.51% Weibull( $\tau$ = 6.90, $\theta$ = 62.76)	
94.58% Loglogistic( $\gamma$ = 2.38, $\theta$ = 1042.25)	-278,879
+ 5.42% Inverse Weibull( $\tau$ = 275.55, $\theta$ = 67.42)	
90.44% Generalized Beta( $\alpha$ = 6.92, $\beta$ = 4.71, $\tau$ = 0.41, $\theta$ = 4209.25)	-278,896
+ 9.56% Gamma( $\alpha$ = 21.36, $\theta$ = 2.73)	
90.44% Generalized Beta( $\alpha$ = 14.47, $\beta$ = 4.64, $\tau$ = 0.21, $\theta$ = 4205.44)	-278,932
+ 9.56% Lognormal( $\mu$ = 4.04, $\sigma$ = 0.21)	



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**Parametric Model Analysis** 

#### Top 5 Parametric Models: Histograms





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#### Top 5 Parametric Models: CTE Comparison to Trans Full Run

Parametric Model	Model CTE	(Model CTE/T. Full Run CTE)*100
90.43% Generalized Beta	CTE70 = 2087.48	104.73
+ 9.57% Weibull	CTE90 = 2583.86	106.74
	CTE99 = 3247.16	103.99
90.49% Beta	CTE70 = 2089.24	104.82
+ 9.51% Weibull	CTE90 = 2574.50	106.36
	CTE99 = 3211.11	103.15
94.58% Loglogistic	CTE70 = 2810.12	140.99
+ 5.42% Inverse Weibull	CTE90 = 4619.34	190.83
	CTE99 = 12,307.37	394.13
90.44% Generalized Beta	CTE70 = 2084.03	104.56
+ 9.56% Gamma	CTE90 = 2578.38	106.52
	CTE99 = 3240.33	103.77
90.44% Generalized Beta	CTE70 = 2113.85	107.72
+ 9.56% Lognormal	CTE90 = 2663.57	107.15
	CTE99 = 3142.91	100.65

#### References

- AMOOF3, https://bitbucket.org/AMOOF3/amoof-3.0/wiki/Home
- Chueh, Y.C.M. 2002. "Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms." North American Actuarial Journal 6(3): 88 - 103
- Chueh, Y.C., and Johnson, P.H. Jr. 2012. "CSTEP: a HPC Platform for Scenario Reduction Research on Efficient Stochastic Modeling - Representative Scenario Approach." Actuarial Research Clearing House 2012.1: 1-12
- Chueh, Y.C., and Johnson, P.H. Jr. 2014. "Case Studies for Model Efficiency: Special Sampling and MLE Bias Correction." *International Journal of Science Commerce and Humanities* 2(1)
- Johnson, P.H. Jr., Chueh, Y.C., and Qi, Yongxue. 2013. "Small Sample Stochastic Tail Modeling: Tackling Sampling Errors and Sampling Bias by Pivot-Distance Sampling and Parametric Curve Fitting Techniques." Actuarial Research Clearing House 2013.1: 1-12



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