Applications of Price Elasticities in Auto Insurance

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Introduction

- Many statistical modeling textbooks concentrate on theoretical concepts and less on data issues and ambiguities that arises in business analysis
- We will look at price elasticity (demand) models and the challenges of applying them to premium dislocation analysis
- Outline of presentation:
  - Business needs for elasticity models
  - Discrete choice model theoretical framework
  - Observation of a “large” premium decrease
  - Applying elasticity models to premium dislocation analysis
  - Elasticity estimates
  - Further issues in modeling elasticities
Elasticity Models - Introduction

- Elasticity (or Demand) Models estimate the change in quantity given a change in price (premium)
  - Quantity Change
    - Renewals/retention ratios for policies-in-force
    - Quote close ratios for new business
  - Premium Change
    - Rate change, aging, stochastic
- Businesses would like to know the effects of rate changes on policy counts and total premiums to assist in strategic planning and forecasting
- Examples are based on 6 month auto policies for 6 states and one rate change in 2013
Theoretical Framework: Discrete Choice Models

- Discrete Choice Analysis is an approach to study consumer choice – given a finite number of choices and their attributes, the model predicts the probability of each individual’s choice.
- We assume the insured has:
  - Two choices – either renew or not; accept quote offer or reject
  - One attribute that varies with choice (premium)
- In estimating models, we use logit link.
Actual Retention by Premium Change

- We generally expect that the higher renewal premium relative to current premium, the lower the expected retention.
- But, that is not true for large premium decreases -- actual retention rates decrease.
- We see this phenomenon appearing across different areas and time periods.

![Graph showing the relationship between log(renewal premium/current premium) and retention rates.](attachment:image.png)
Application: Premium Dislocation

• Premium dislocation analysis compares the before and after premiums due to a rate change assuming the same policy characteristics

• This appears to be an ideal application for elasticity models in estimating impact of a rate change

• Timing lags are challenges to our current process:
  • Detailed policy characteristics datasets are created only twice a year
  • Takes a long time to validate retention – e.g. for annual policies, need to wait two years after the rate plan effective date to capture the full policy year impact

• Example: Apply renewal model to a state rate filing in May 2013
  • Validate by calculating estimated renewal rate deciles and compare with actual renewal rates
  • Premium dislocation data is as mid year 2012
Application: Model Validation

- Above charts are based on policies that came up for renewal after the rate change and were used in the premium dislocation analysis.
- The model performs well on actual premiums but not so well on dislocation data.
Application: Premium Dislocation

- The above premium ratios are fairly uncorrelated:

<table>
<thead>
<tr>
<th>Renewal Prem/Current Prem</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>1.01</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Prem Dislocation</td>
<td>1.04</td>
<td>0.03</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Elasticity Definitions

- Elasticity (%Δ quantity/%Δ price)
  - Arc Elasticity: \( \frac{(Q_2-Q_1)/[(Q_1+Q_2)/2]}{(P_2-P_1)/[(P_1+P_2)/2]} \)
  - Elasticity (Discrete): \( \frac{(Q_2-Q_1)/Q_1}{(P_2-P_1)/P_1} \)
  - Point Elasticity: \( \frac{\partial Q}{Q}/\frac{\partial P}{P} = \frac{\partial \ln(Q)}{\partial \ln(P)} \)

- Calculate elasticity at the policy or quote level
- Elasticity varies across policies (given premium)
- Elasticity also depends on premium level (given policy)

- Retention elasticity estimates below are based on auto six month policies for six states
## Elasticity Estimates

### Point Elasticity (current prem)
- Mean: -0.28
- Standard Deviation: 0.20
- Mean: -0.96
- Standard Deviation: 0.50

### Arc Elasticity (90% current & current)
- Mean: -0.24
- Standard Deviation: 0.17
- Mean: -0.91
- Standard Deviation: 0.49

### Arc Elasticity (current & 110% current)
- Mean: -0.32
- Standard Deviation: 0.22
- Mean: -0.93
- Standard Deviation: 0.50

### Elasticity (90% current & current)
- Mean: -0.22
- Standard Deviation: 0.16
- Mean: -0.81
- Standard Deviation: 0.43

### Elasticity (current & 110% current)
- Mean: -0.30
- Standard Deviation: 0.20
- Mean: -0.84
- Standard Deviation: 0.44

- New business is several times more elastic than retention
- Elasticity’s variance is fairly large
- Elasticity depends on particular definition
Further Issues

- Handle 6 and 12 month policies together (survival analysis, discrete logistic regression)
- Distinguish between different sources of premium changes (identify premium changes coming from deterministic [e.g. aging], stochastic [e.g. violations], and rate changes)
- Calculating competitor premiums need many assumptions to run a vendor’s program
- Frequency in updating models (e.g. retention validation on a rate change takes long time)
Summary

• Business knowledge is needed to develop and apply models useful to business needs. Challenges include:
  • Define business data fields in terms of modeling statistical variables
  • Adapt to business’ data processes and limitation
  • Communicating methodology and results to non-technical audiences

• Elasticity results
  • Retention less elastic than new business quotes
  • Auto less elastic than homeowners (not shown here)
Reference


Appendix – Binary Model

Assume 2 possible choices and V is linear in parameters with logit link:

\[ V_{in} = \beta_1 x_{in1} + \beta_2 x_{in2} + \ldots + \beta_k x_{ink} \]

\[ V_{jn} = \beta_1 x_{jn1} + \beta_2 x_{jn2} + \ldots + \beta_k x_{jnk} \]

\[ P_{in} = \exp(V_{in})/\left[\exp(V_{in})+\exp(V_{jn})\right] \]

\[ = \exp(\beta'x_{in})/\left[\exp(\beta'x_{in})+\exp(\beta'x_{jn})\right] \]

\[ = 1/(1 + \exp(\beta'(x_{in}-x_{jn})) \]

Example: let i=1=CSAA IG; j=2=Competitor, n is insured
Appendix – Point Elasticity

Elasticity Calculation:

\[ E(P_{in},x_{ink}) = (1-P_{in})\beta_{k} \cdot \partial V_{in} / \partial x_{ink} \cdot x_{ink} \]
\[ E(P_{in},x_{jnk}) = -P_{in}\beta_{k} \cdot \partial V_{in} / \partial x_{jnk} \cdot x_{jnk} \]

In our example, \( V_{in} = \ln(x_{in}) \), so

\[ E(P_{in},x_{ink}) = (1-P_{in})\beta_{k} \]
\[ E(P_{in},x_{jnk}) = -P_{in}\beta_{k} \]

\( i = \text{choice}, \ n = \text{insured}, \ k = \text{variable in model} \)
Appendix – Auto Models

• Important Variables in Auto Retention/Renewal Model:
  • Renewal premium ratio, competitor premium ratio, multipolicy discount, state, age of oldest insured, auto persistency

• Important Variables in Auto Quote Model:
  • Competitor premium, policy term, number of drivers, number of vehicles, bi limit, payment method, median age