Predator-Prey Dynamics: A Model for Competition, Regulation, and Strategic Planning

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Rick Gorvett, FCAS, ASA, CERA, MAAA, ARM, FRM, PhD
Director, Actuarial Science Program
State Farm Companies Foundation Scholar in Act. Sci.
University of Illinois at Urbana-Champaign
Updated Title of Talk

• Agent-based modeling (ABM)
  – Still relevant, but not part of this talk
  – Leave it to others

• Competition
  – P-P modeling framework is at least as relevant for regulation, strategic planning, etc.
Personal Approach to ARC

• Past talks
  – Complex adaptive systems
  – Behavioral economics
  – Hyperbolic discounting
  – Predator-prey dynamics

→ Interesting potential areas of application to actuarial science / risk management
Background

• Complex adaptive systems
  – Built bottom-up from individual agents
  – Self-organization and emergence
  – Irreducible

• Evolutionary processes
  – Parallels between economic systems and biological evolutionary theory
    • Complex systems, self-organization, emergence, adaptation (natural selection)
Background (cont.): Economics and Biology

“The precise mathematical relationship which describes the link between the frequency and size of the extinction of companies, for example, is virtually identical to that which describes the extinction of biological species in the fossil record. Only the timescales differ.”

- Ormerod (2005)
Background (cont.): “Business Ecosystem”

- Economic community
- Interacting organizations and individuals
- Customers and competitors
- Leadership companies (“the keystone species”)
  - Influence the co-evolutionary processes
  (Moore, 1996)
Rothschild (1990): “a capitalist economy can best be comprehended as a living ecosystem. Key phenomena observed in nature – competition, specialization, co-operation, exploitation, learning, growth, and several other – are also central at business life.”
Background (cont.):

Personal

• Personal perspectives
  – Practicing actuary
  – Interdisciplinarity
  – Strategic and operational

• Interest in modeling economic / financial / insurance phenomenon
  – Competition
  – Supply / demand
  – Underwriting cycles
  – Regulatory effects
Lotka-Volterra Equations
(two-species model)

• Let: \( X \): size of **prey** population

\( Y \): size of **predator** population

• Differential equation system:

\[
\frac{dX}{dt} = X(a - bY) = aX - bXY
\]

\[
\frac{dY}{dt} = -Y(c - dX) = -cY + dXY
\]
Lotka-Volterra Equations (cont.)

- where: \( a, b, c, d > 0 \)

\( a = \) intrinsic prey growth rate

\( b = \) predation rate coefficient

\( c = \) predator mortality rate

\( d = \) predator reproduction rate per prey
Lotka-Volterra Equations (cont.)

- Assumptions underlying (two-species) L-V model:
  - Prey have access to an inexhaustible food supply
  - Prey increase exponentially in absence of predators
  - Predators feed only on prey (and thus will starve in the absence of prey)
  - No limit to amount of prey consumed by predators
  - Environment is homogeneous through time
Sample Predator-Prey Time Chart
Assuming No Predators

- **Population Size**
  - 1,000,000.00
  - 9,000,000.00
  - 800,000.00
  - 7,000,000.00
  - 600,000.00
  - 500,000.00
  - 400,000.00
  - 300,000.00
  - 200,000.00
  - 100,000.00
  - 0.00

- **Time**

- **Prey**
- **Predators**
Sample Predator-Prey Time Chart
Assuming No Prey
L-V: More Realistic

• Allow for self- (intra-) species competition

• Updated differential equation system:

\[
\frac{dX}{dt} = X(a - bY - eX) = aX - bXY - eX^2
\]

\[
\frac{dY}{dt} = -Y(c - dX + fY) = -cY + dXY - fY^2
\]
Related: The Logistic Equation

- Single process or system through time
- Differential equation:

\[
\frac{dW}{dt} = gW - hW^2
\]

- \( g \): reflects net birth-death rate
- \( h \): reflects competition under limited resources
Some Applications of These Models

• Prey – predator dynamic relationship:
  – Rabbits – foxes
  – Competitors in a market
  – Total product demand in a market
  – Economic fluctuations (real wages and employment)
  – Oil prices – marginal production / demand destruction / energy policy
Potential Applications in Act Sci / Ins / Risk Mgt

• Competition amongst insurers
• Pricing of insurance / other risk management vehicles
• Underwriting cycle
• Regulation

→ Part of CAE Grant: U/G Research