Pricing Non-Recourse Provisions and Mortgage Insurance for Joint-Life Reverse Mortgages Considering Mortality Dependence: a Copula Approach

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Outline

- Introduction and Literature Review
- The Valuation Framework for HECM Program
  - Non-recourse provision
  - Mortgage insurance
- Modeling Joint-Life Mortality
  - Copula model
- Numerical Analysis
- Conclusion

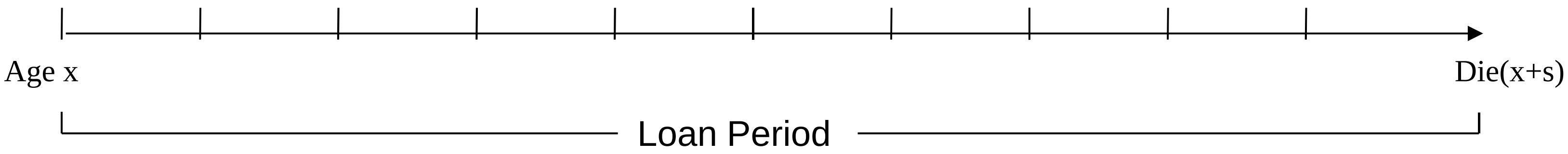
Introduction

What are Reverse Mortgage Products?

- A kind of home equity conversion that allows the elder persons to borrow money with their home as the collateral .
- The loans accrue interest are only repaid once the people is died or leave the house. →No Fixed Maturity Date
- Loan value is determined by borrower's age, property value and interest rate
- For example: a rolled-up mortgage (Lump-Sum)

Loan Value: K ---> $K_t = Ke^{vt}$ at time t

Property Value: H_0 ---> H_t

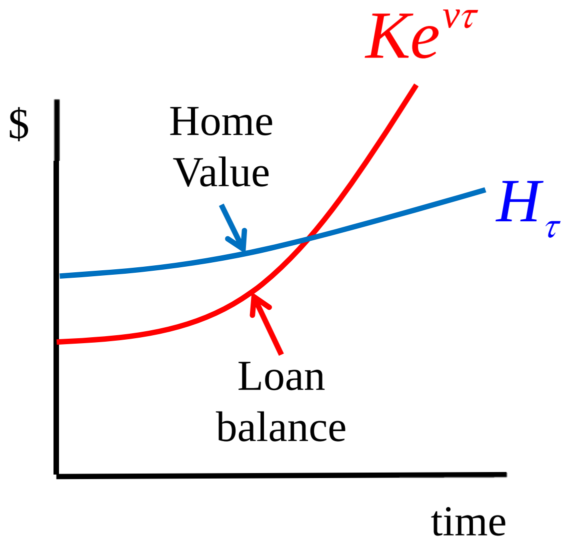


The Risk from Lender Prospective

Key risk

Key risk	
Longevity risk	If a borrower lives a longer time than the expected lifespan that may lead the loan balance above the sale proceed of the property.
Property risk	If the house price grows at a lower rate than expected, the loan balance may exceed the home value. Lenders may suffer from the losses.
Interest rate risk	The rise of interest rates increases the possibility of non-repayment when the loan eventually terminates.

$$Ke^{v\tau} > H_\tau$$

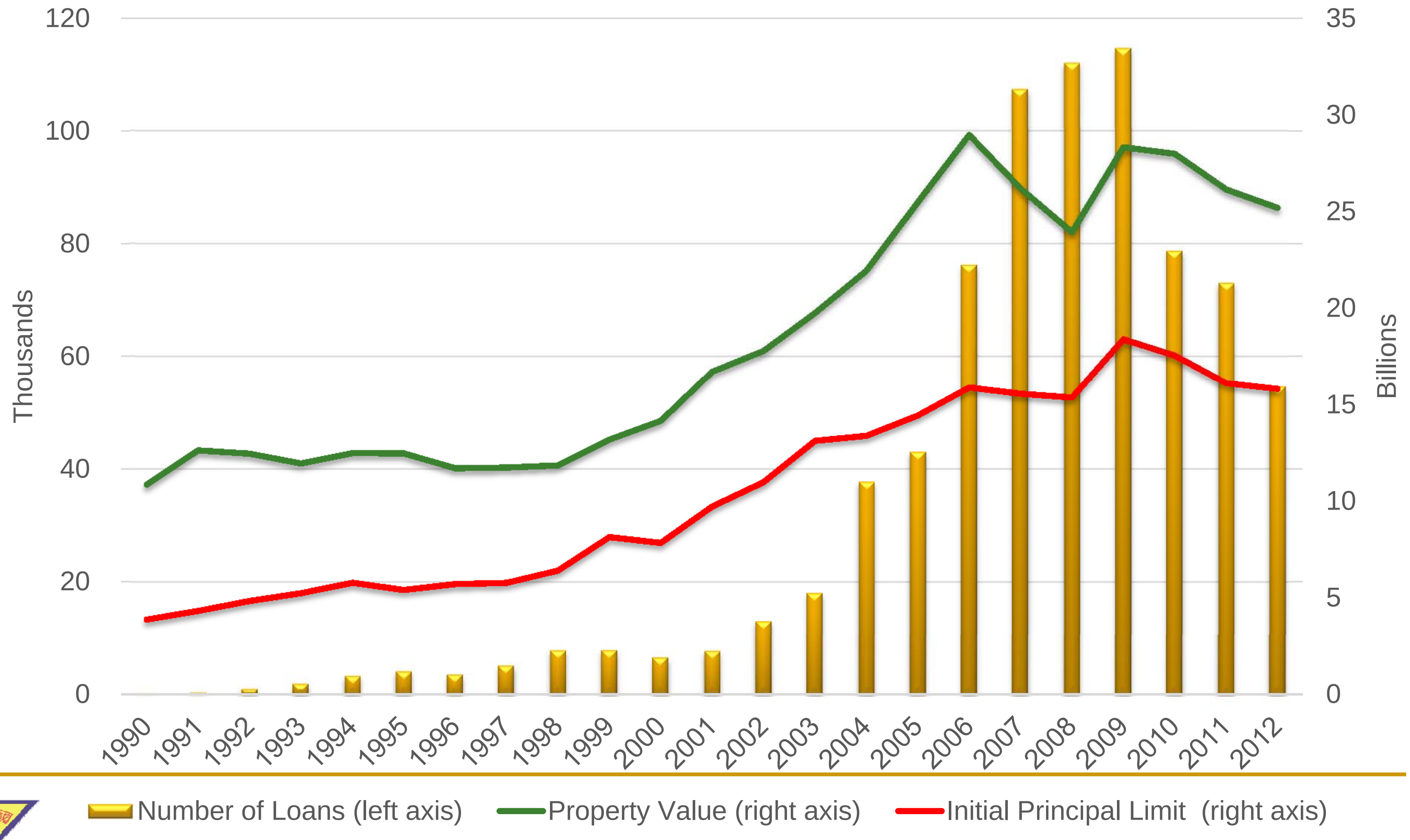


The RM Market: in the U.S.

- Home Equity Conversion Mortgage (HECM) program first introduced in 1989.
- After financial crisis, the private reverse mortgage market has evaporated so that HECM loans represent nearly 100% of newly originated reverse mortgages.(Shen, REE 2011)
- The HECM loan is a non-recourse debt.

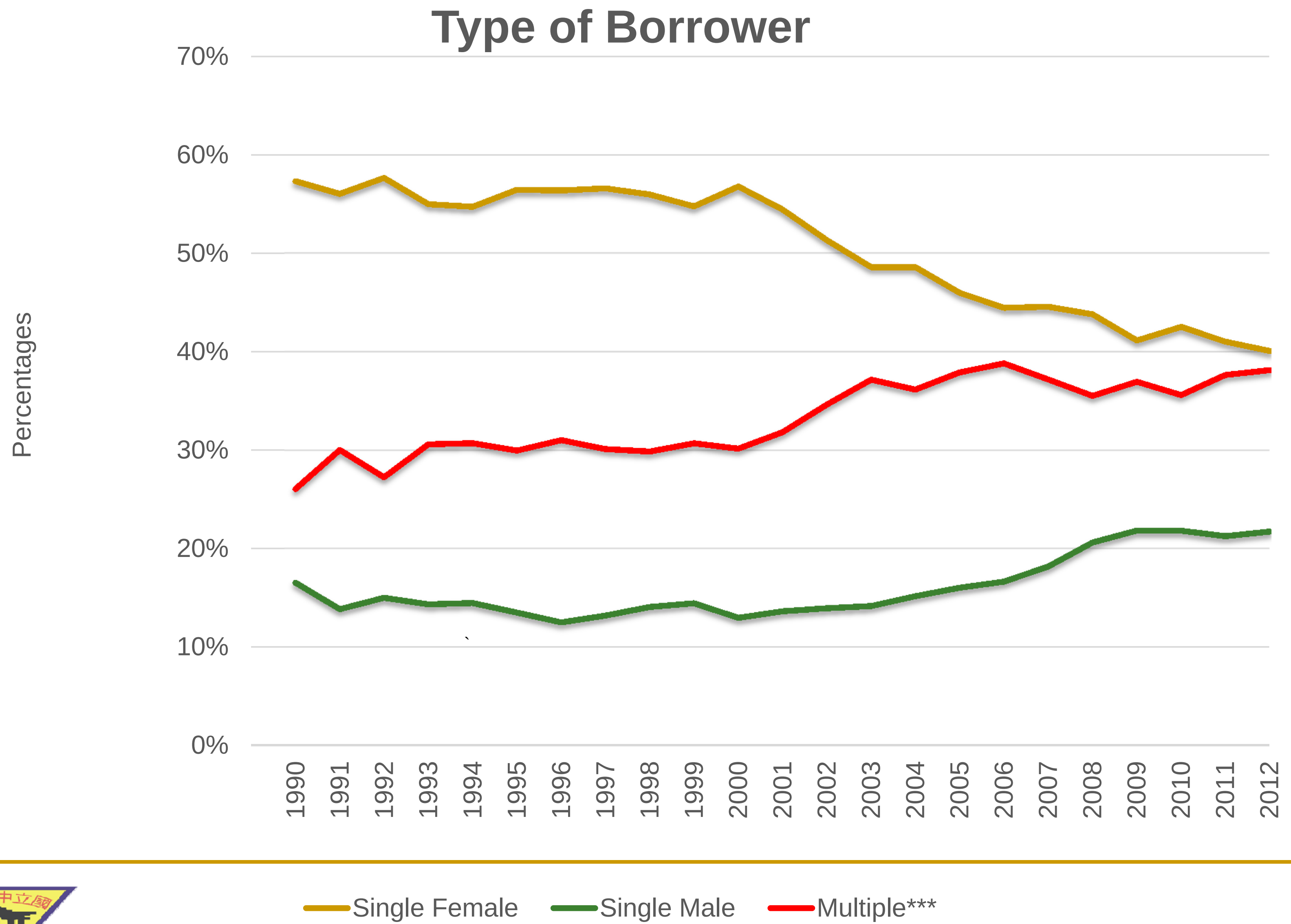
- Some factors drive the need for RM
  - Globally Increasing Life Expectancies
  - Many elders are considered to be “Cash Poor & Equity Rich”.

HECM loan



120
100
80
60
40
20
0
Thousands
35
30
25
20
15
10
5
0
Billions
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
Number of Loans (left axis)
Property Value (right axis)
Initial Principal Limit (right axis)

HECM Loan



Type of Borrower
70%
60%
50%
40%
30%
20%
10%
0%
Percentages
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
Single Female
Single Male
Multiple***

Literature

- Most of the existing literatures of RMs focus on the following issues:
- Single-life RMs
  - Except for Chia and Tsui(2004) work on joint-life RMs
  - Assume mortality is Independent.
- Lump-Sum payment
  - Except for Lee et al.(2013) work on tenure RMs
- Underlying House Price Dynamic
  - Szymamoski(1994) uses the GBM model.
  - Li et al.(2010) propose the ARMA-EGARCH model
  - Chen et al. (2010) propose ARMA-GARCH model.

Motivation of this research

- Joint life RMs are getting more and more popular in HECM loans.
- Shemyakin and Youn (1999) point out three possible sources of association between husbands' and wives' mortality.

  *common lifestyle

  *common disaster

  *broken-heart factor

Purpose of this research

- To capture the mortality dependence for pricing joint-life RMs
  - Lump Sum vs. Tenure

- Pricing Non-Recourse Provisions (NRP) and Mortgage Insurance Premiums (MIP)

- Investigate the effect of mortality dependence on NRP and MIP

Pricing Framework for Joint-life Revere Mortgages

Payoff of the non-recourse provision

- The cash flow of the borrower can be written as follows:

$$\text{repayment} = \begin{cases} -H_t & H_t < L_t \\ -L_t & H_t \geqq L_t \end{cases} = -L_t + \max(L_t - H_t, 0)$$

  - where L_t is the outstanding balance of the loan and H_t is the value of the mortgaged property at a time t.
- Claim loss function at time t :

$$CL_t = \max\,(L_t - H_t, 0)$$

Payoff of the non-recourse provision

- The fair value of the non-recourse provision (NRP) written on a cohort group aged *x* can be expressed as the present value of total expected claim losses on a cohort group aged *x* as follows:

$$\text{PV ECL} = \sum_{t=0}^{\omega - x - 1} {}_{t|1}q_{\overline{xy}} . E_Q[e^{-rt} \max(L_t - H_t, 0)]$$

where

- r is the risk-free interest rate
- ${}_{t|1}q_{\overline{xy}}$ is the probability that both a male aged x and a female aged y will survive another t year, but die before next year.
- E_Q is the expectation under the risk-adjusted measure Q.

Outstanding Balance

- At time t, the house value and the accumulated loan balance are

$$H_t = H_0 e^{Y_t}$$

$$L_t = 0.02H_0 \cdot e^{ut} + \sum_{j=1}^{t} e^{u(t-j)} 0.005 L_j + \sum_{j=0}^{t} A \cdot e^{u(t-j)}$$

  - where A is annuity value(unknown).

Mortgage insurance premium

- The present value of mortgage insurance premium of reverse mortgage can be calculated as

$$\text{PV MIP} = 0.02\text{H}_0 + \sum_{t=1}^{\infty} {}_{\text{t}}p_{\overline{xy}} . e^{-rt}(0.005 L_t)$$

- Initial premium: 2% of the property value
  Continuous premium: 0.5% of outstanding balance

- What does the ratio of PVMIP/NRP mean?

Modeling Joint-Life Mortality

Independent Joint-life Mortality

- We denote the marginal survival functions by S_x^m and S_y^f, so far all $t \geq 0$

$$ {}_t p_x^m = S_x^m = \Pr[T^m(x) > t] $$

$$ {}_t p_y^f = S_y^f = \Pr[T^f(y) > t] $$

Joint survival probability notation

$$ {}_t p_{xy} = S_{xy}(t,t) = \Pr[T^m(x) > t \text{ and } T^f(y) > t] \underset{\text{indep}}{=\!=} S_x^m(t) S_y^f(t) $$

$$ {}_t p_{\overline{xy}} = 1 - {}_t q_x^m \cdot {}_t q_y^f = {}_t p_x^m + {}_t p_y^f - {}_t p_{xy} $$

$$ {}_{t|1} q_{\overline{xy}} = {}_t p_{\overline{xy}} - {}_{t+1} p_{\overline{xy}} $$

Correlated Joint-life Mortality

- Shemyakin and Youn (1999) utilized copula functions to calculate joint survival probability for pricing joint life insurance and list three possible sources of association between husbands' and wives' mortality.

  *common lifestyle

  *common disaster

  *broken-heart factor

Correlated Joint-life Mortality

- The joint survival probability can be written as

$$ {}_t p_{xy} = \Pr[T^m(x) > t \text{ and } T^f(y) > t] \underset{\text{dep}}{=} C(S_x^m(t), S_y^f(t); \theta) $$

  where θ is correlated parameter.
- The joint survival probability which the couple are survival exceeding one year can be written as $p_{x+t:y+t} = S_{(x+t)(y+t)}(1,1) = C(p_{x+t,}\, p_{y+t}; \theta_t)$
- Combing above equation

$$ {}_t p_{xy} = p_{xy} \cdot p_{x+1:y+1} \cdots p_{x+t-1:y+t-1} = \prod_{i=0}^{t-1} C(p_{x+i}\,,\ p_{y+i};\ \theta_t) $$

Copula Functions

- Gaussian copula(Normal copula):

$$C(u_1, u_2;\theta) = \Phi_G(\Phi^{-1}(u_1), \Phi^{-1}(u_2);\theta)$$

- Student-t copula:

$$C(u_1, u_2;\theta_1, \theta_2) = t_{\theta_1,\theta_2}\{t_{\theta_1}^{-1}(u_1), t_{\theta_1}^{-1}(u_2)\}$$

- Clayton copula:

$$C(u_1, u_2;\theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}}$$

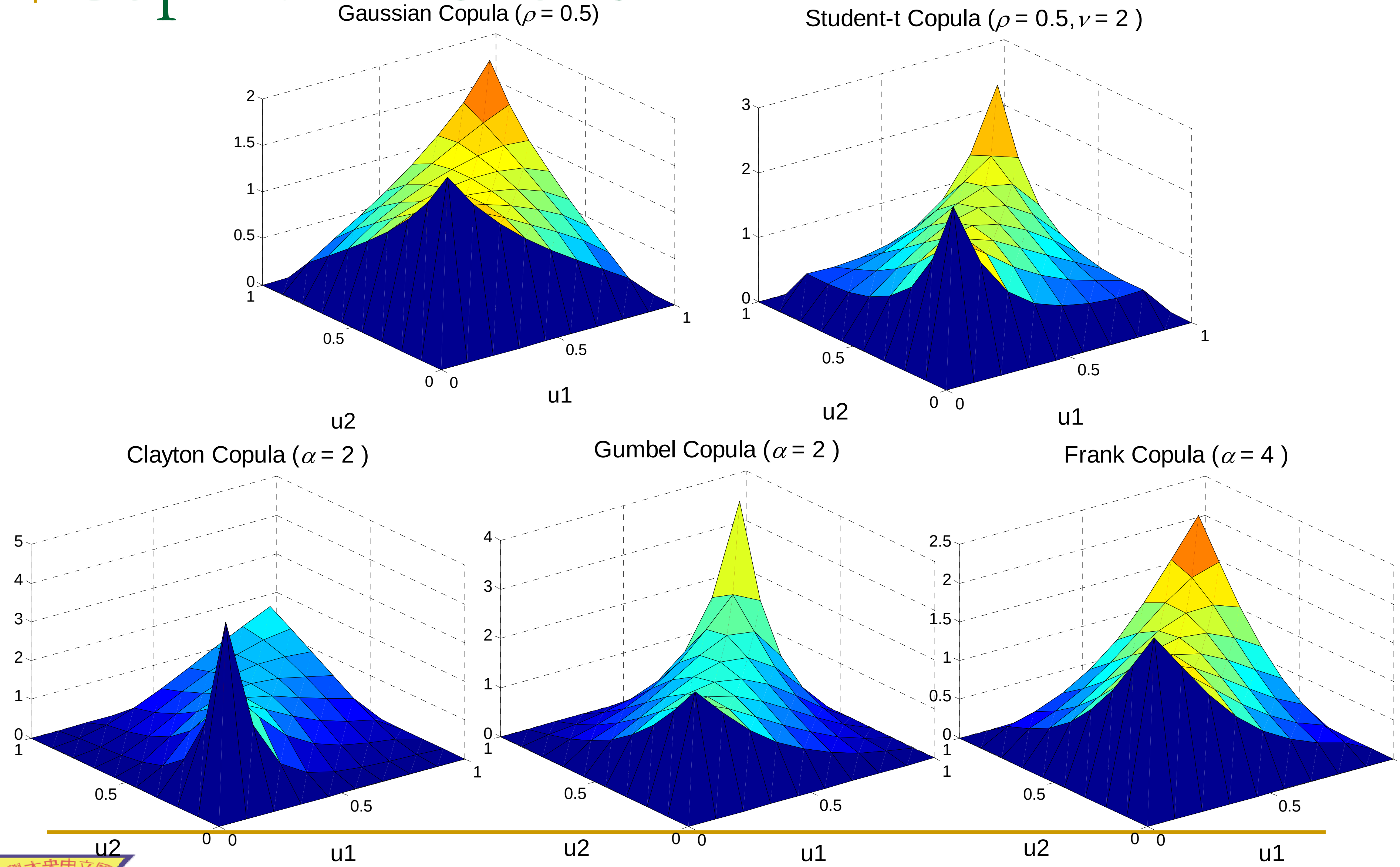
- Frank copula:

$$C(u_1, u_2;\vartheta) = -\frac{1}{\vartheta}\ln[1 + \frac{(e^{-\vartheta u_1} - 1)(e^{-\vartheta u_2} - 1)}{e^{-\vartheta} - 1}]$$

- Gumbel copula:

$$C(u_1, u_2;\theta) = \exp[-\{(-\log u_1) + (-\log u_2)\}^{-\frac{1}{\theta}}$$

Copula Functions



Lee-Carter Model with Cohort Effect

- Renshaw and Haberman (2006) made cohort-based extension to the LC model.
- $\ln(m_{xtc}) = \alpha_x + \beta_x(t)\kappa_t + \beta_x(c)\kappa_c^* + \varepsilon_{xt}$
  - m_{xt} : central death rate for a person aged *x* at time *t*
  - α_x : describes the average age-specific mortality
  - β_x : decline in mortality at age *x*
  - κ_t : represents the general mortality level
  - κ_c^* : cohort effect
  - $\varepsilon_{x,t}$: deviation of the model from the observed log-central death rates

Modeling the House Price Dynamics

Specifically, the ARMA-GARCH model can be denoted as a ARMA(P, Q) series

$$y_t = c + \sum_{p=1}^{P} b_p y_{t-p} + \sum_{q=1}^{Q} a_q \varepsilon_{t-q} + \varepsilon_t$$

where P is the order of the autocorrelation terms, Q is the order of the moving average terms, b_p is the pth-order autocorrelation coefficient, a_q is the qth-order moving average coefficient.

Furthermore, white noise, $\varepsilon_{t,j}$, is assumed Gaussian distribution with mean 0 and variance denoted by the GARCH(R, M) model. The model as follows

$$\sigma_{t,j}^2 = \delta_{0j} + \sum_{r=1}^{R} \delta_{rj} \varepsilon_{t-r,j}^2 + \sum_{m=1}^{M} \beta_{mj} \sigma_{t-m,j}^2$$

where R is the order of the GARCH terms, M is the order of the ARCH term, r is the rth-order GARCH coefficient, m is the mth-order ARCH coefficient, $\delta_{rj} > 0$ for r = 1, …, R and $\beta_{mj} > 0$ for m = 1, …, M.

Risk-adjust Probability

- House Price Dynamic
  - Esscher transform (Buhlmann et al.(1996)
  - Lee et al.(2010), Li et al.(2010), Chen et al.(2010)

- Mortality Dynamic
  - Wang Transform

To change the probability measure from the real-world to a risk-neutral measure, Wang (2000) proposes a distortion operator:

$$F_n^{\tau}(x) = \Phi\left(\Phi^{-1}\left(F_n(x)\right) + \tau\right), \tag{18}$$

  - Lin and Cox (2005), Lee et al.(2010), Yang et al.(2013)

$$\xi_T = \prod_{t=\Delta t}^{T} \frac{\exp(\varphi(t) Y(t))}{E_P(\exp(\varphi(t) Y(t)) | F_{t-\Delta t})}.$$

Then, we define a new martingale measure Q_φ by

$$\left.\frac{dQ_\varphi}{dP}\right|_{F_T} = \xi_T.$$

Under the risk-neutral measure Q_φ, the housing return becom

$$Y(t) = \ln\left(\frac{H(t)}{H(t-\Delta t)}\right)$$
$$= r(t-\Delta t)\,\Delta t - \frac{1}{2}h(t) + \varepsilon_H^Q(t),$$

Empirical Study

Fitting House price dynamics

- The house price data

  *Quarterly House Price Index (HPI)

  *1991Q1~2011Q3

→ ARMA(2,2)-GARCH (1, 1)

Mortality Dynamics

- The mortality rate data

  *U.S., Male and Female

- Data range

  *Age: 60~99

  *Year: 1950~2007

- Source

  *Human Mortality Database(HMD)

  *http://www.mortality.org/

Fitting model (Male)

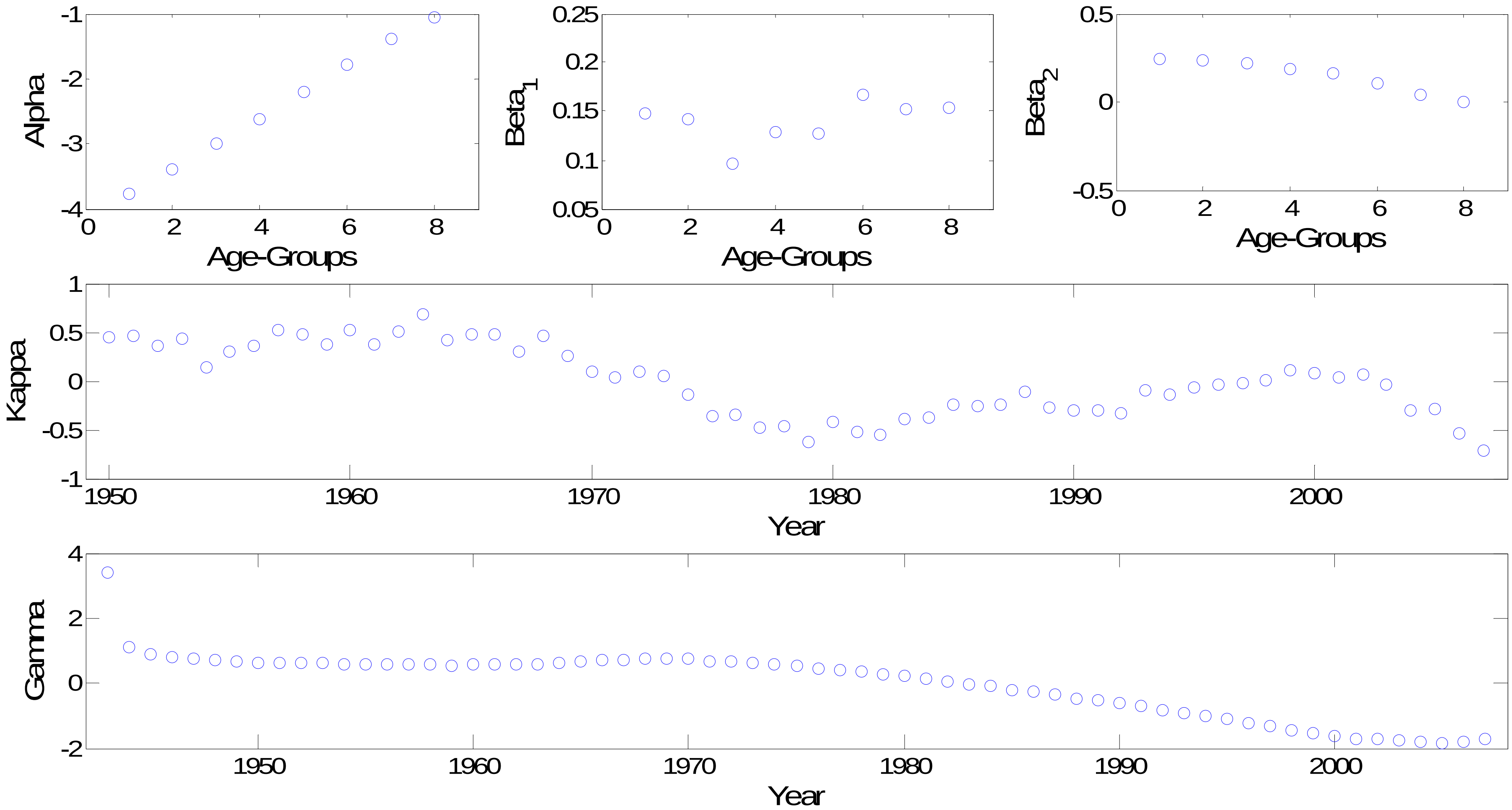


Figure: Lee-Carter model with cohort estimated parameters

Fitting model (Female)

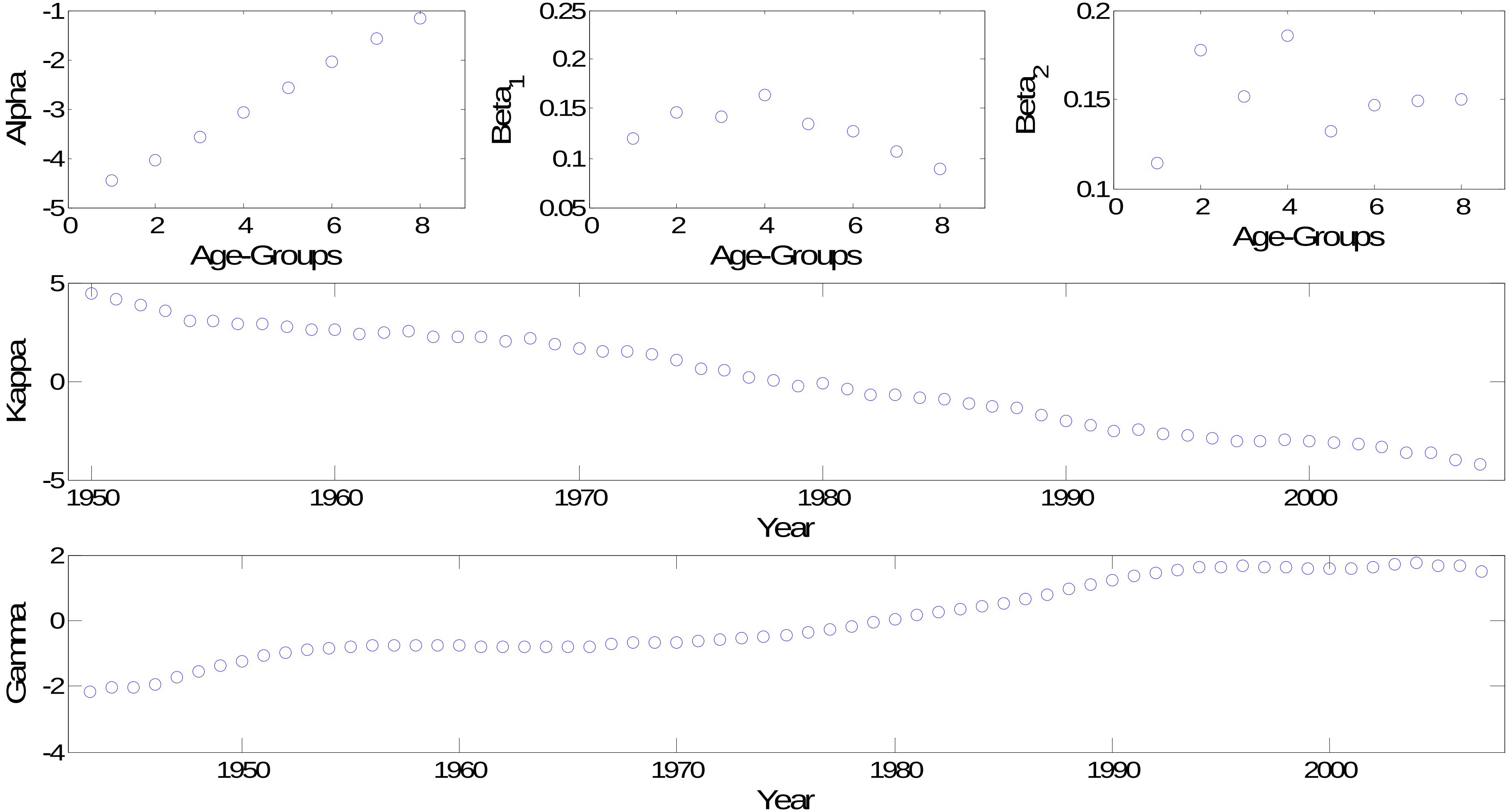
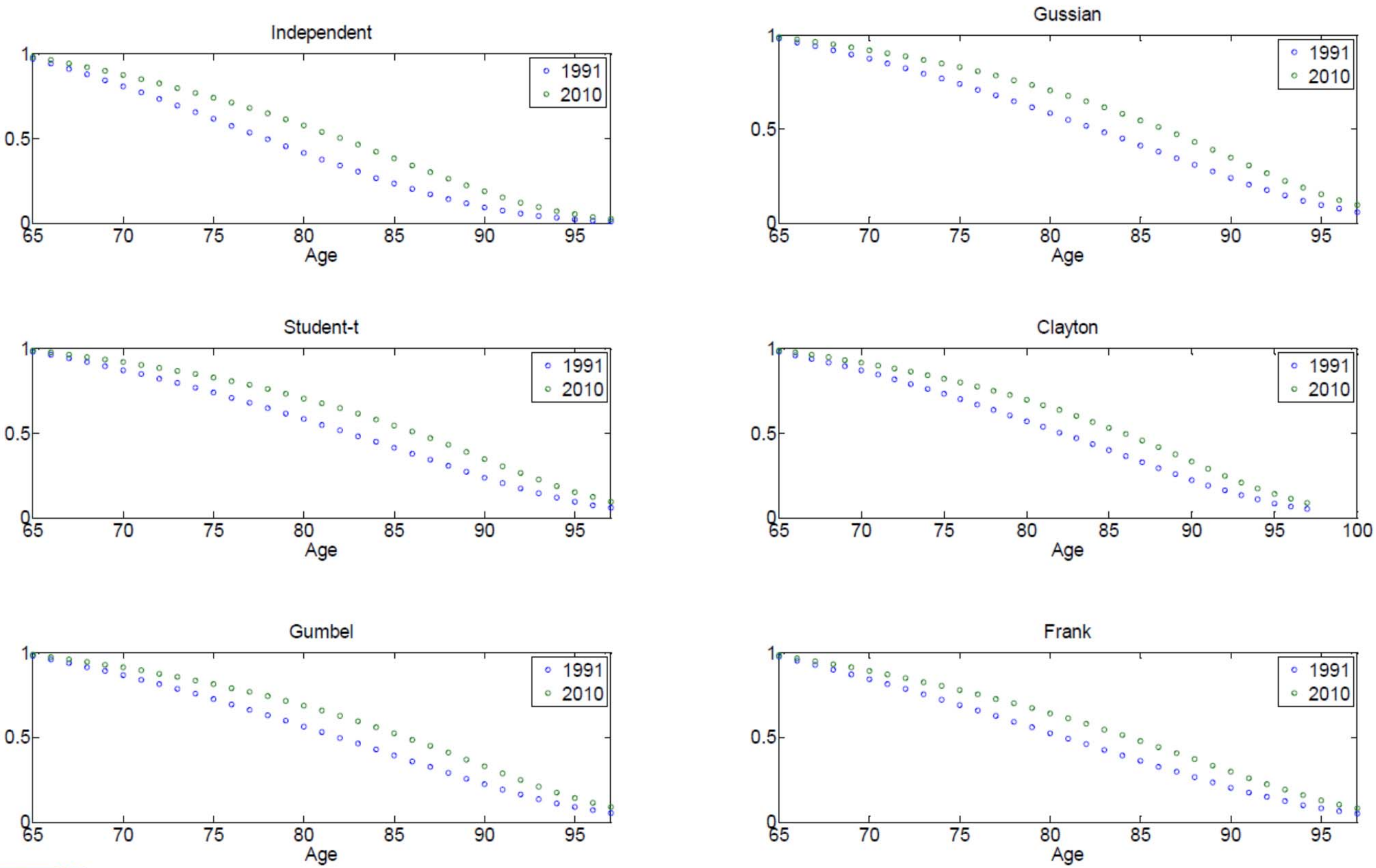


Figure: Lee-Carter model with cohort estimated parameters

Selection of Copula Models

- Comparing with AIC and BIC, the Clayton and Gumbel copulas are selected for difference ages.
  - Clayton copula is selected for 60~72 age and Gumbel copula is selected for 73~95 age.
- Trivedi and Zimmer (2005) suggested, when correlation between spouses' age at death is strongest in the left tail of the joint distribution, Clayton is an appropriate modeling choice.

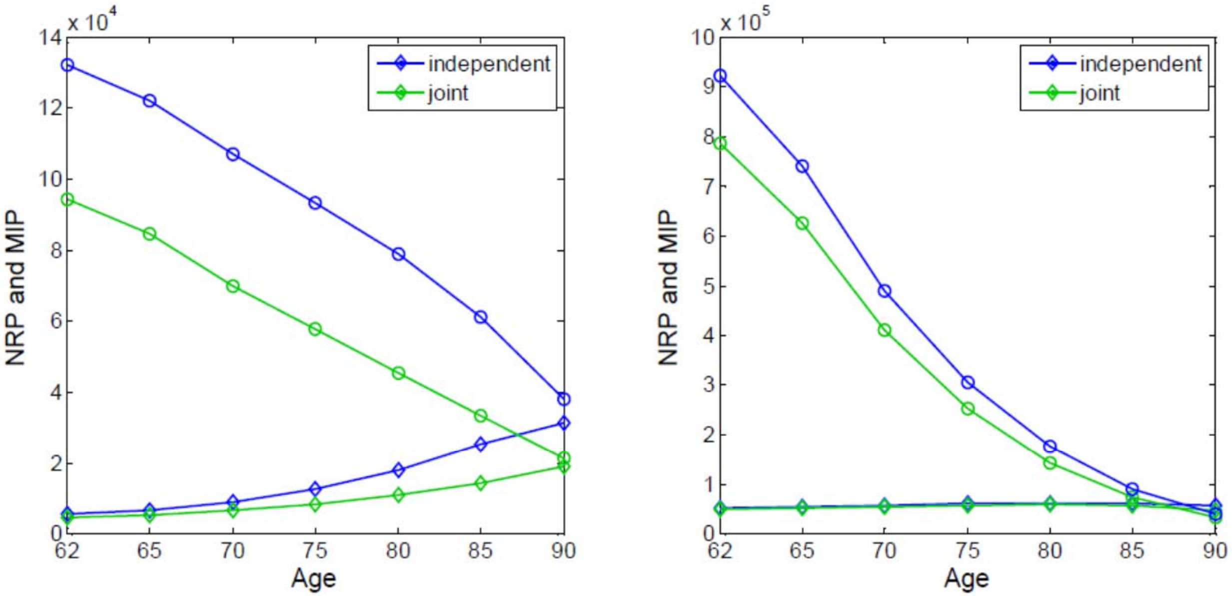
Joint-life Probability at same age(xx)



Assumptions for Calculating NRP and MIP

- Assume house price independent mortality rate.
- Male and Female are at the same age.
- H_0 =300,000
- r =1.7523%.
- u =2.451%.
- g =2%.
- κ =6%.
- ω =99.
- We use Monte Carlo simulation 10,000 times and utilize Antithetic variance reduction to reduce the variation of pricing the reverse mortgage.

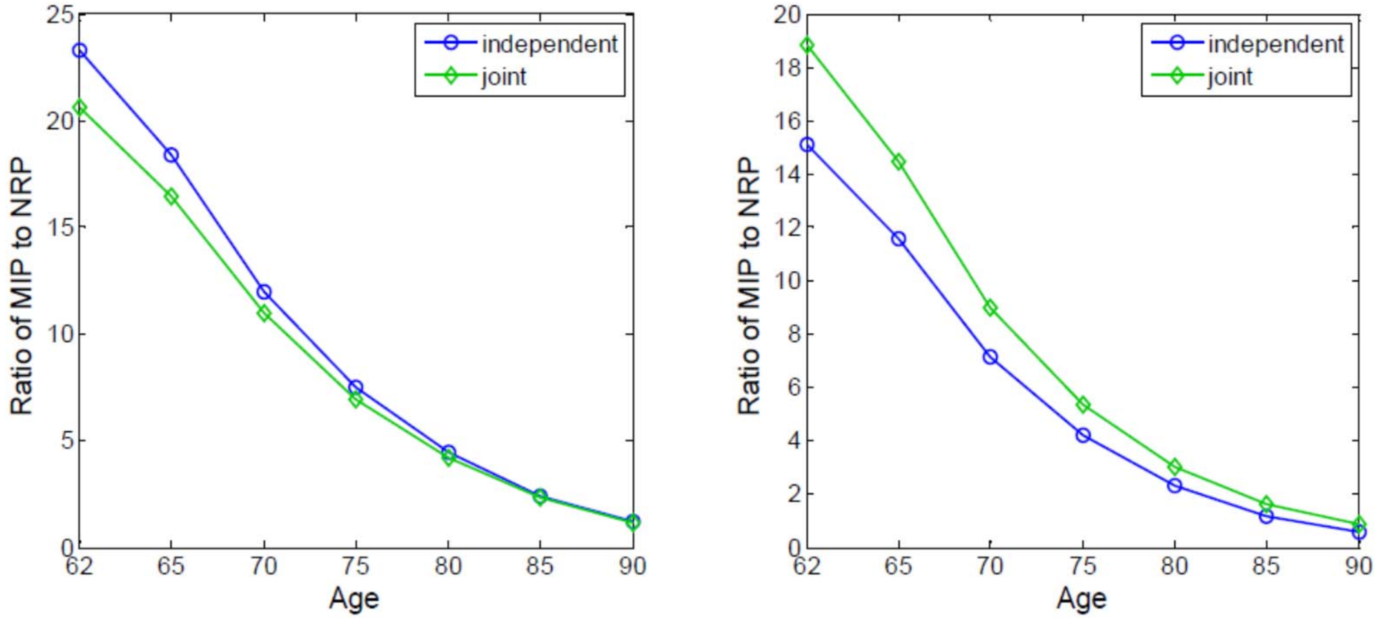
NRP and *MIP* : lump sum and tenure payment



Note: the –○– lines and –◇ – lines are the present values of mortgage insurance premiums (*MIP*) and the value of the non-recourse provision (*NRP*), respectively.

Figure 7: Values of the *NRP* and *MIP* under products of lump sum (right figure) and tenure payment (left figure).

MIP/NRP : lump sum and tenure payment



Note: the –○– lines and –◇ – lines are the present values of mortgage insurance premiums (*MIP*) and the value of the non-recourse provision (*NRP*), respectively.

Figure 8: Ratios of *MIP* to *NRP* under products of lump sum (right figure) and tenure payment (left figure).

Initial house price value

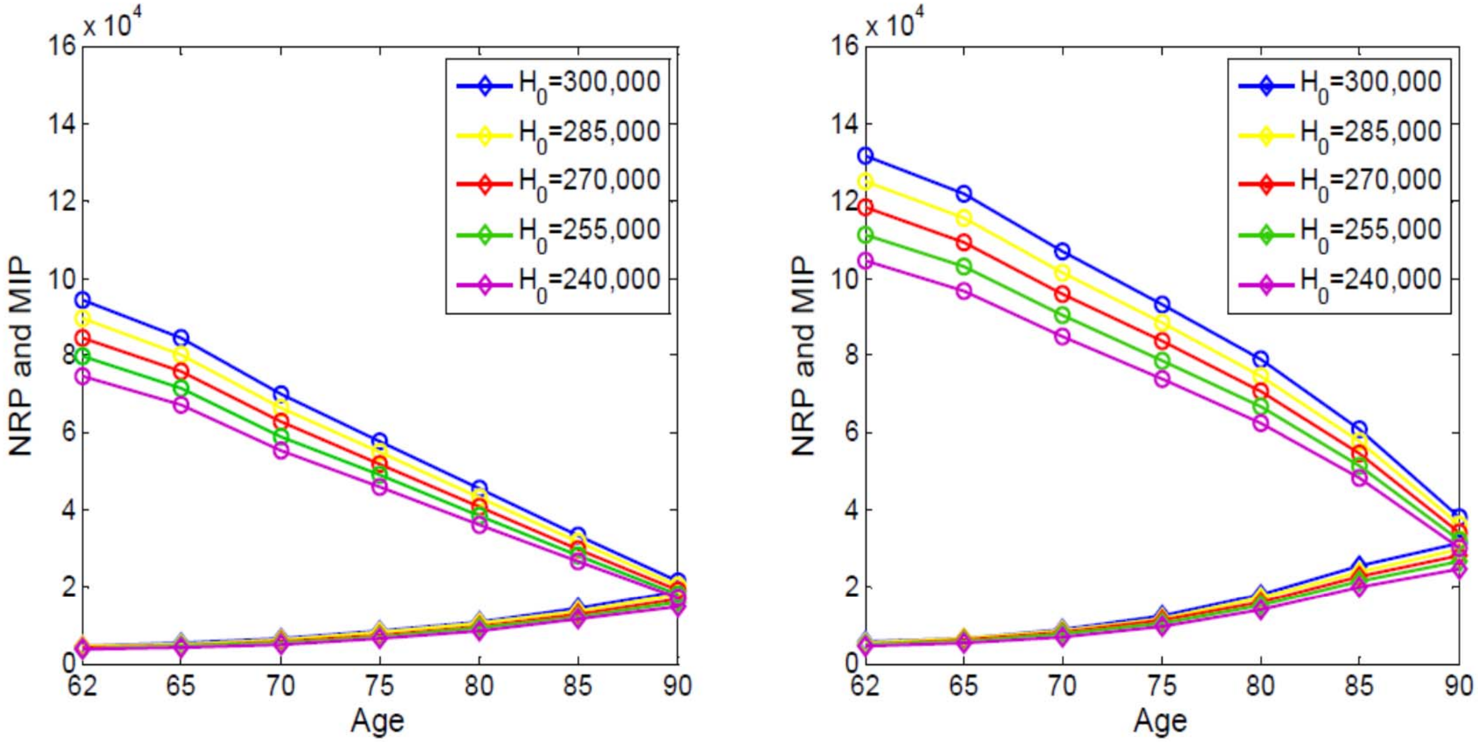


Figure 10: Values of the NRP and MIP under independent situation (right figure) and copula function (left figure) in 2010.

Initial house price value

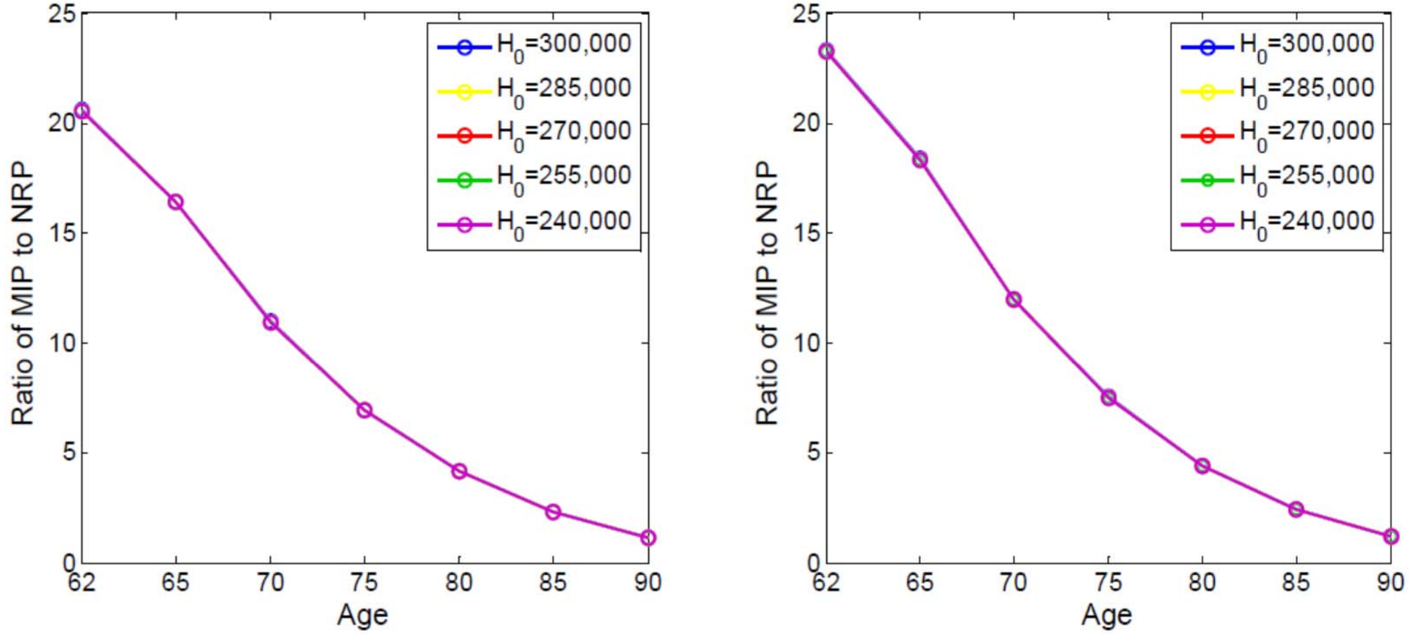


Figure 12: Ratios of *MIP* to *NRP* under independent situation (right figure) and copula function (left figure) in 2010.

Conclusion

- Ignoring the dependence between joint-life mortality overestimates the NRP and MIP
  - Such effect is more significant for lump sum RMs
- The insurance company can use their own mortality experience to replace the empirical analysis.
  - Frees et al.(1996)
  - Luciano, et al. (2008) model the mortality risk of couples of individuals, according to the stochastic intensity approach.

Thank you!

Q&A