Interplay of Asymptotically Dependent Insurance Risks and Financial Risks

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Outline

1. Question we consider

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2. Model:
   - A discrete time model for an insurer
   - Insurer's initial wealth: $W_0 = x$; wealth at time m: W_m
   - Insurer's net insurance loss within the m-th period: X_m
   - Insurer's investment return in the m-th period: R_m
   - Insurer's wealth process:

$$W_m = W_{m-1} R_m - X_m = x \prod_{j=1}^{m} R_j - \sum_{i=1}^{m} X_i \prod_{j=i+1}^{m} R_j \tag{1}$$

3. Comments

Insurance risks

- Losses from insurance claims
- One-claim-causes-ruin phenomenon (Embrechts et al. (1997), Muermann (2008), etc.)

1992 Hurricane Andrew

- Insured loss: $16 billion
- More than 60 insurance companies became insolvent (Muermann (2008, *NAAJ*))

2004 Indian Ocean Earthquake and Tsunami

- Damaged about $15 billion
- Not much insurance loss due to lack of insurance coverage

2005 Hurricane Katrina

- Insured loss: $41.1 billion
- Damaged $108 billion

2011 Japan Earthquake, Tsunami and Nuclear Crisis

- Insured loss: $14.5-34.6 billion

2012 Hurricane Sandy

- Insured loss: $19 billion
- Damage: over $68 billion

Insurance risks

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- Hedging
- Heavy-tailedness: assumption of regular variation

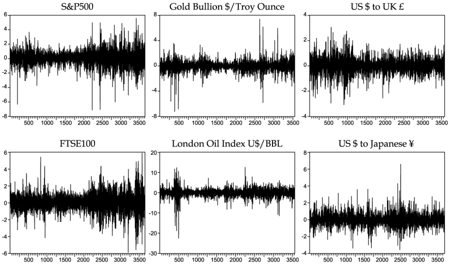
Insurance risks

- Losses from insurance claims
- One-claim-causes-ruin phenomenon (Embrechts et al. (1997), Muermann (2008), etc.)
- Hedging
- Heavy-tailedness: assumption of regular variation
- A distribution function F is said to have a regularly varying tail with index $\alpha > 0$, written as $\overline{F} \sim \mathcal{R}_{-\alpha}$, if

$$\lim_{x\to\infty} \frac{\overline{F}(xt)}{\overline{F}(x)} = t^{-\alpha}, \qquad t > 0.$$

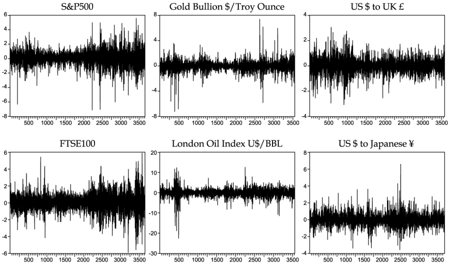
- Examples: Pareto, t-distribution, Burr distribution
- Losses due to earthquakes: $0.6 < \alpha < 1.5$
  Losses due to hurricanes: $1.5 < \alpha < 2.5$. (See, e.g. Ibragimov et al. (2009))

Financial risks



Daily log-returns for the period of 01/03/1989 to 06/30/2003. (Angelidis and Degiannakis (2005))

Financial risks



Daily log-returns for the period of 01/03/1989 to 06/30/2003. (Angelidis and Degiannakis (2005))

Sylized facts on returns from stocks/stock indices (Basrak et al. (2002), Rachev et al. (2005), Kelly and Jiang (2014), etc.):

- Regularly varying with tail index $2 < \alpha < 4$
- Asymmetric

Q: What are the effects of these risks on the survival of the insurer?

Related studies

1. Related studies:
   - Nyrhinen (1999)
   - Tang and Tsitsiashvili (2003)
   - ⋮
   - Chen (2011)
   - Fougères and Mercadier (2012)
2. Comments. Why asymptotic dependence?

(Asymptotic/Extreme) dependence

- Copula of (X, Y): $C(\cdot, \cdot)$
- Survival copula of (X, Y): $\hat{C}(\cdot, \cdot)$
- Asymptotic dependence

$$\lim_{u \downarrow 0} \frac{\hat{C}(u, u)}{u} = \lim_{u \downarrow 0} \frac{\Pr\left(X > F_X^{\leftarrow}(1-u), Y > F_Y^{\leftarrow}(1-u)\right)}{u} > 0$$

- Examples
- Assume asymptotic dependence for $(X, 1/R) = (X, Y)$

Assumptions

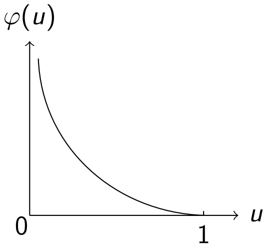
1. (X_1, R_1), (X_2, R_2), $\ldots$ are i.i.d. copies of (X, R).
2. Suppose that $\overline{F_X} \in \mathcal{R}_{-\alpha}$, $\alpha > 0$, with $F_X(-x) = o\left(\overline{F_X}(x)\right)$. Also suppose that the distribution of R has a regularly varying tail at 0 with index $\beta > 0$.
3. Suppose that there exists some function $H(\cdot, \cdot)$ on $[0, \infty]^2 \backslash \{\mathbf{0}\}$, such that $H(t_1, t_2) > 0$ for every $(t_1, t_2) \in (0, \infty]^2$ and

$$\lim_{u \downarrow 0} \frac{\hat{C}\left(ut_1, ut_2\right)}{u} = H(t_1, t_2) \quad \text{on } [0, \infty]^2 \backslash \{\mathbf{0}\}. \tag{2}$$

Example

Let (X, Y) have an Archimedean copula

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)). \tag{3}$$



Assume that the generator φ satisfies

$$\lim_{u \downarrow 0} \frac{\varphi(1 - tu)}{\varphi(1 - u)} = t^{\theta}, \qquad t > 0, \tag{4}$$

for some constant $\theta > 1$ (Note that $\theta \geq 1$ if (4) holds). (See Charpentier and Segers (2009))

Example: $\varphi(u) = (-\ln u)^{\theta}$

Then (2) holds with

$$H(t_1, t_2) = t_1 + t_2 - \left(t_1^{\theta} + t_2^{\theta}\right)^{1/\theta} > 0, \quad (t_1, t_2) \in (0, \infty]^2. \tag{5}$$

Implications

1. Let $Y = 1/R$ and $Y_i = 1/R_i$, $i = 1, 2, \ldots$. Assumption 1 $\implies$ Y is regularly varying with index $\beta > 0$.
2. X and Y are asymptotically dependent.
3. The random vector $\left(1/\overline{F_X}(X), 1/\overline{F_Y}(Y)\right) \in MRV_{-1}$ (multivariate regularly varying), and the vague convergence

$$x \Pr\left(\frac{1}{x}\left(\frac{1}{\overline{F_X}(X)}, \frac{1}{\overline{F_Y}(Y)}\right) \in \cdot\right) \xrightarrow{v} \nu(\cdot) \qquad \text{on } [0,\infty]^2 \backslash \{\mathbf{0}\} \tag{6}$$

holds with the Radon measure ν defined by

$$\nu\left[\mathbf{0}, (t_1, t_2)\right]^c = \frac{1}{t_1} + \frac{1}{t_2} - H\left(\frac{1}{t_1}, \frac{1}{t_2}\right), \qquad (t_1, t_2) \in (0,\infty)^2. \tag{7}$$

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Implications cont'd

The random vector (X, Y) follows a nonstandard MRV structure; i.e., for some Radon measure μ, the following vague convergence holds:

$$x \Pr\left(\left(\frac{X}{b_X(x)}, \frac{Y}{b_Y(x))}\right) \in \cdot\right) \xrightarrow{v} \mu(\cdot) \qquad \text{on } [0,\infty]^2 \backslash \{\mathbf{0}\}, \tag{8}$$

where $b_X(x) = \left(\frac{1}{\overline{F_X}}\right)^{\leftarrow}(x)$ and $b_Y(x) = \left(\frac{1}{\overline{F_Y}}\right)^{\leftarrow}(x)$.

See Resnick (2007).

Probabilities of ruin

- Finite-time horizon:

$$
\begin{aligned}
\psi(x;n) &= \Pr\left(\min_{1\leq m\leq n} W_m < 0 \,\middle|\, W_0 = x\right) \\
&= \Pr\left(\min_{1\leq m\leq n}\left(x\prod_{j=1}^{m} R_j - \sum_{i=1}^{m} X_i \prod_{j=i+1}^{m} R_j\right) < 0\right) \\
&= \Pr\left(\max_{1\leq m\leq n}\sum_{i=1}^{m} X_i \prod_{j=1}^{i} Y_j > x\right)
\end{aligned}
$$

- Infinite-time horizon:

$$
\psi(x) = \Pr\left(\max_{1\leq m<\infty}\sum_{i=1}^{m} X_i \prod_{j=1}^{i} Y_j > x\right)
$$

Main result 1

Theorem 2.1

Under Assumptions 1–3 we have

$$\psi(x;n) \sim \Pr\left(\sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j > x\right) \sim \left(\sum_{i=0}^{n-1}\left(\mathrm{E}\left[Y^{\alpha\beta/(\alpha+\beta)}\right]\right)^{i}\right)\frac{v(A)}{b^{\leftarrow}(x)},$$

where the set $A = \{(t_1, t_2) \in [0,\infty]^2 : t_1^{1/\alpha} t_2^{1/\beta} > 1\}$, *the function* $b(\cdot) = \left(1/\overline{F_X}\right)^{\leftarrow}\left(1/\overline{F_Y}\right)^{\leftarrow}(\cdot)$, *and the measure* v *is defined by relation (7).*

Note: $\psi(\cdot;n) \in \mathcal{R}_{-\alpha\beta/(\alpha+\beta)}$

Main result 1 cont'd

Theorem 2.2

In addition to Assumptions 1–3, assume that $\mathrm{E}\left[Y^{\alpha\beta/(\alpha+\beta)}\right] < 1$. *Then*

$$\psi(x) \sim \frac{1}{1 - \mathrm{E}\left[Y^{\alpha\beta/(\alpha+\beta)}\right]} \frac{v(A)}{b^{\leftarrow}(x)},$$

On the estimation of ν. See, e.g., Resnick (2007), Nguyen and Samorodnitsky (2013).

Asymptotic dependence vs asymptotic independence

Roughly, under (asymptotic) independence, the probability of ruin decays faster.

Examples: Under corresponding conditions,

- $\psi(x; n) \sim C_n\overline{F_X}(x)$ (Chen (2011))
- $\psi(x; n) \sim C_n\overline{F_X}(x) + D_n\overline{F_Y}(x)$ (Li and Tang (2014))

Main result 2 (& role of regulation)

- Prudent Person Investment Principle (PPIP) under Solvency II: no requirement on what insurers can invest and what they can not, but they are encouraged to reduce their investment in equities (take less investment risks) due to high capital charges, which would reduce the overall profit.
- If the insurer only invests a proportion $\pi < 1$ of its wealth into risky assets, and the rest earns a risk free return $r_f \geq 1$, then $R > (1-\pi)r_f$, which would violate Assumption 2.
- Assumption 2^*: Suppose that $\overline{F_X} \in \mathcal{R}_{-\alpha}$, $\alpha > 0$, with $F_X(-x) = o\left(\overline{F_X}(x)\right)$. Also suppose that R is bounded below by some positive number r.

Main result 2

Theorem 2.3

Under the Assumptions 1, 2^*, and 3, we have

$$\psi(x;n) \sim \Pr\left(\sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j > x\right) \sim \left(\sum_{i=0}^{n-1} \left(\mathrm{E}\left[Y^\alpha\right]\right)^i\right) r^{-\alpha}\overline{F_X}(x). \tag{9}$$

Theorem 2.4

In addition to the assumptions of Theorem 2.3, assume that $\mathrm{E}\left[Y^\alpha\right] < 1$. Then

$$\psi(x) \sim \frac{1}{1-\mathrm{E}\left[Y^\alpha\right]} r^{-\alpha}\overline{F_X}(x). \tag{10}$$

Concluding Remarks

- Asymptotically dependent insurance risks and financial risks both play a significant role in affecting the insurer's survival.
- By regulating insurers' investment behavior, like discouraging too risky investments, regulators can help significantly reduce their probability of ruin.

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Thank you!