Optimal Capital Budgeting and Risk Management with Shared and Dependent Risks

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Six points I want to make in this talk:

- 1. The big picture is that there are often shared and dependent uncertainties among different business divisions in large financial institutions, therefore it is important to model these dependent risks for ERM.
- 2. Copulas models dependency and allows for flexible marginal distributions.
- 3. The dependent decision tree approach simply models the dependent uncertainties based on copulas in a tree structure, so that it is easier to incorporate the managerial flexibilities.
- 4. The ERM framework is basically an optimization problem.
- 5. We use the state of nature tree to reduce dimensionality for computational simplicity.
- 6. The hypothetical example I'll give will illustrate that ERM provides extra value when incorporating managerial flexibilities and hedging strategies in the proposed framework.

Motivations

- Enterprise risk management (ERM) help companies optimize operational decisions.
- Risk management and capital budgeting are two critical components of the dynamic corporate decision process. They naturally connected by the dependent risk exposures and a variety of other synergetic relationships within an intricate corporate structure
- It is challenging is to fully encompass the two components into the overall corporate decision making.

The challenges of an intricate corporate structure

  – Multiple divisions, dependent risks, dynamic planning
  – Intertwined enterprise decisions (on capital budgeting, RM, financing, compensation, etc.)

- Risk dependency modeling in corporate decisions
  – Shared and dependent risks exist across business divisions/ projects and time periods
  – The inter-dependencies entail nontrivial interplay of corporate decisions

Literature Review

- Capital budgeting
  - Limited risk dependency modeling, albeit recognition of the importance
    - Tsetlin and Winkler (2005)
    - Gustafsson and Salo (2005)
    - Meiner, Christofides, and Salkin (2001)
  - Agency problems
    - E.g., Harris and Raviv (1998)
- Corporate/Enterprise risk management
  - Managing risks holistically rather than in "silos"
    - Ai et al. (2012)
  - "Coordinate" corporate functions
    - Froot et al. (1993) and Froot and Stein (1998)

Literature Review

- Capital budgeting
  - Modeling the internal allocation process
    - The NPV standard (Graham and Harvey 2001, Graham et al. 2010)
  - Agency problems
    - “Centralized” vs. “Delegated” (Marino and Matsusaka 2005)

Overview

Develop an integrated framework that allows to design optimal investment and risk management strategies jointly and endogenously

- Characterize an optimal decision making problem with

–Evaluations of capital requirements, cash flow potentials, and risk exposures first within each division

–Corporate level optimal decisions in a multi-division, multi-project, multi-period environment

–Fully accounting for dependent risks across business divisions and time periods

–Simultaneously obtaining optimal risk management strategies

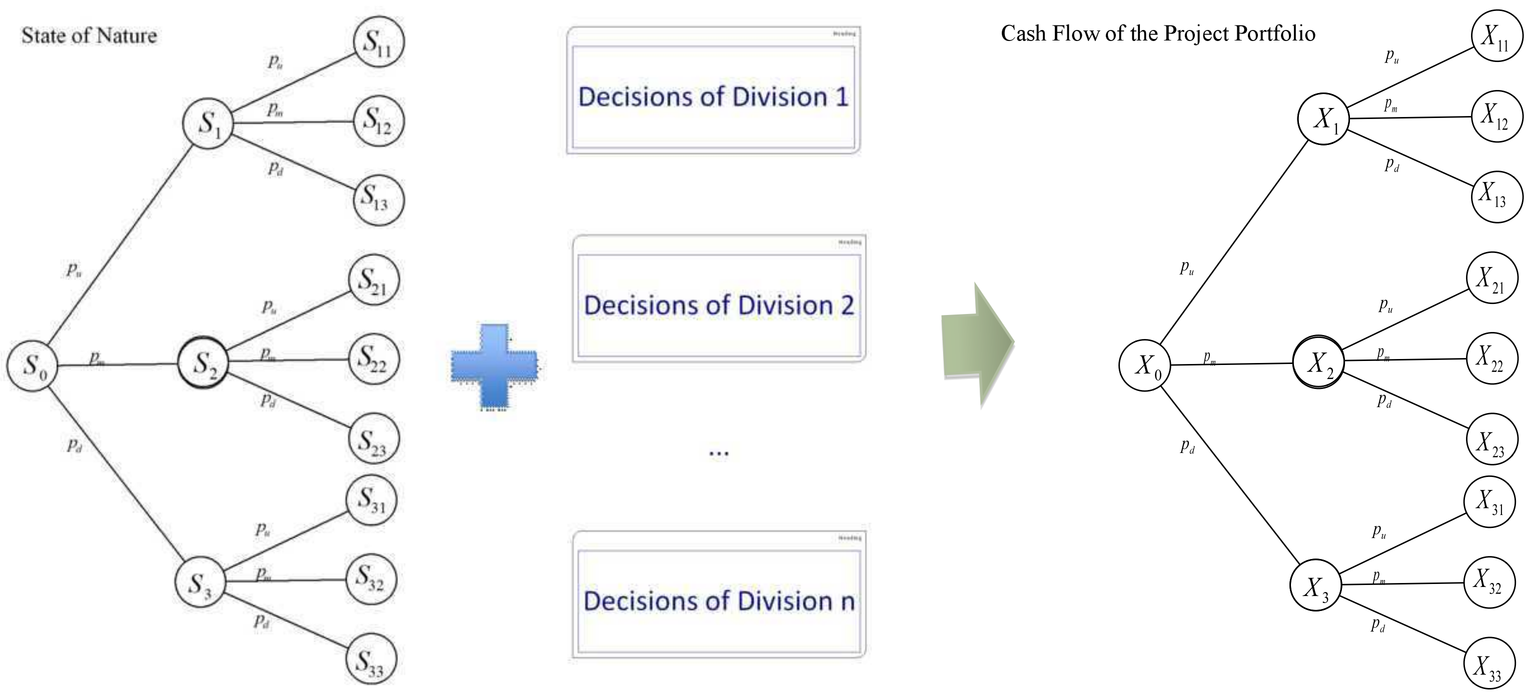
- Uses the intuitive interface of decision tree as an auxiliary step

Model Set-Up

- Solve a capital allocation optimization problem of the corporate decision maker
- Model structure
  - Objective function: Maximize her appropriate utility function
  - Corporate “state of nature” tree: probability tree structure to capture dependent risk within and across divisions and periods, using the copula model
  - Characterize cash flows of the corporate project portfolio with the tree structure and a set of constraints
    - Logical consistency
    - Capital resource
    - Risk
  - Incorporate RM strategies by recognizing cost and adjusting future cash flow positions

Model Set-Up

- Mapping decisions onto corporate state of nature tree



Model Set-Up

- Two recent techniques to facilitate the modeling process in the decision tree analysis
  - Copula-based dependency modeling (Wang and Dyer 2012)
    - Represent dependent risks with arbitrary marginal distributions and a wide variety of copula dependency structure with a sequence of conditional distributions
    - Each risk realization is represented with a trinomial discrete approximation by the extended Pearson-Tukey method (Keefer and Bodily 1983)
  - A dimension reduction technique (Gustafsson and Salo 2005)
    - Directly map decisions onto the state of nature tree instead of branching out a new set of decision variables
    - Size of the decision tree = size of the state tree
    - By way of properly constructed constraints

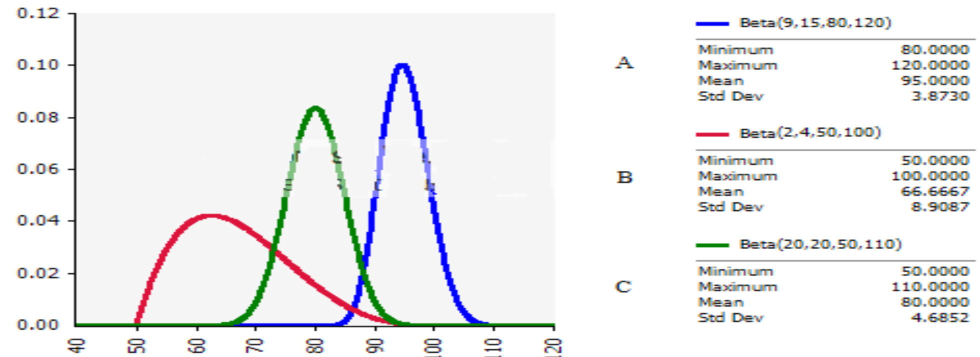
An Illustrative Example

- Consider a financial services company with a two-period planning horizon
  - Two business divisions: loan and insurance, each with one potential project to invest in
  - In period 1, both are subject to market risk
  - In period 2, loan division is subject to credit risk and insurance division is subject to actuarial pricing risk
  - These risks are dependent
  - The CEO makes investment decisions at the corporate level, given an initial capital budget, by maximizing her expected utility from the final financial outcomes at the end of the two-period horizon

An Illustrative Example

- Distributional and dependency assumptions
  - We use scaled beta distributions for its flexibility

Risk
A (market risk factor: revenues)
B (credit risk: loan losses)
C (actuarial risk: underwriting loss)



  - Correlation Structure among Risks

	A	B	C
A	1	-0.5	-0.7
B	-0.5	1	0.5
C	-0.7	0.5	1

  - Considers a Normal copula

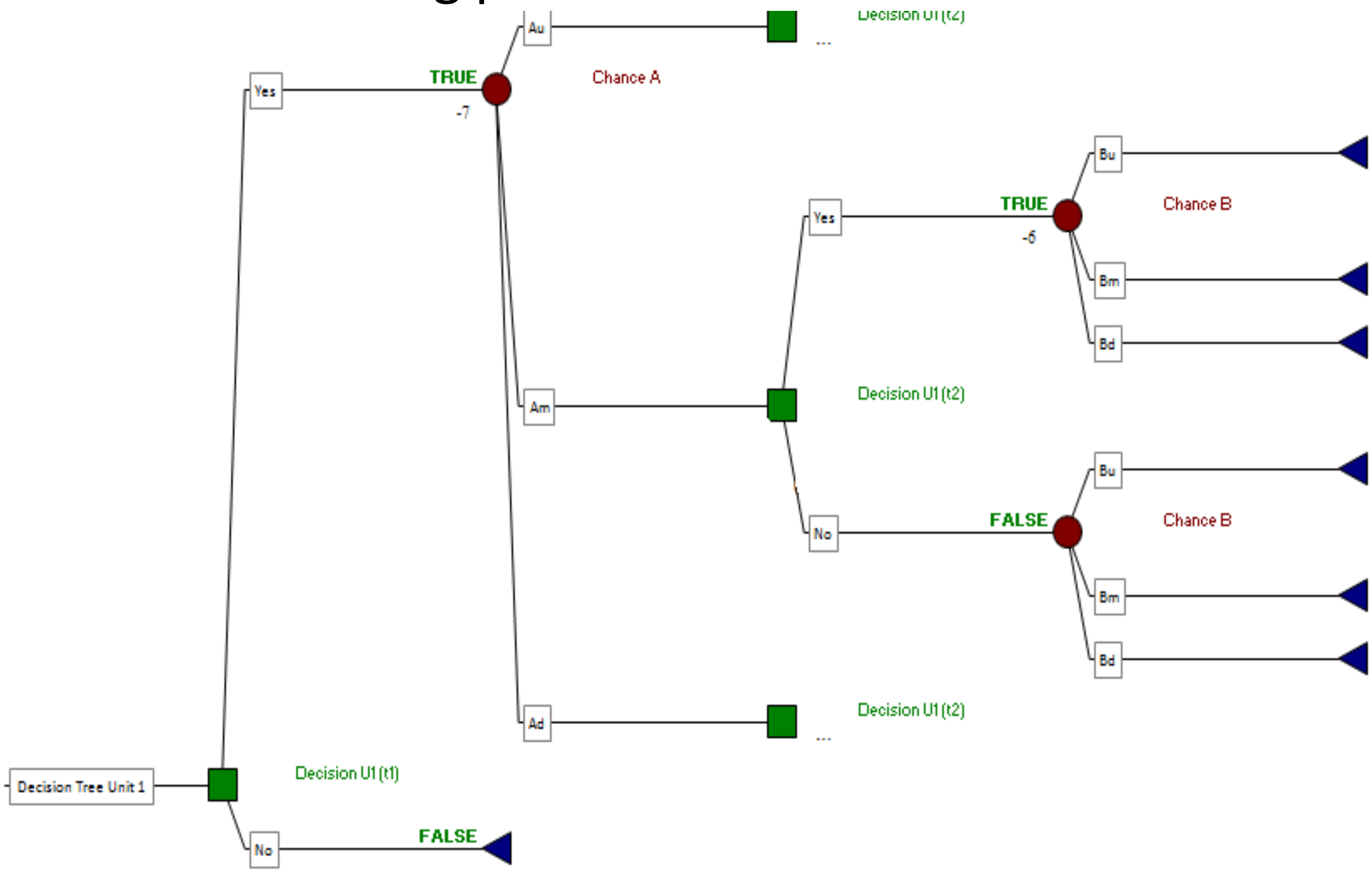
An Illustrative Example

- Characterize cash flows of the corporate project portfolio
  - Investment requirements, risk-free interest rate, etc.

Decision For	Investment ($ million)	Notation
Division 1 (t=0)	$Inv^{1,0} = 7$	$X^{1,0,Y}$, $X^{1,0,N}$
Division 1 (t=1)	$Inv^{1,1} = 6$	$X^{1,k,Y}$, $X^{1,k,N}$, k=1,2,3
Division 2 (t=0)	$Inv^{2,0} = 7$	$X^{2,0,Y}$, $X^{2,0,N}$
Division 2 (t=1)	$Inv^{2,1} = 6$	$X^{2,k,Y}$, $X^{2,k,N}$, k=1,2,3
Initial Resources (I^0)	30	
Risk-Free Interest Rate	6%	

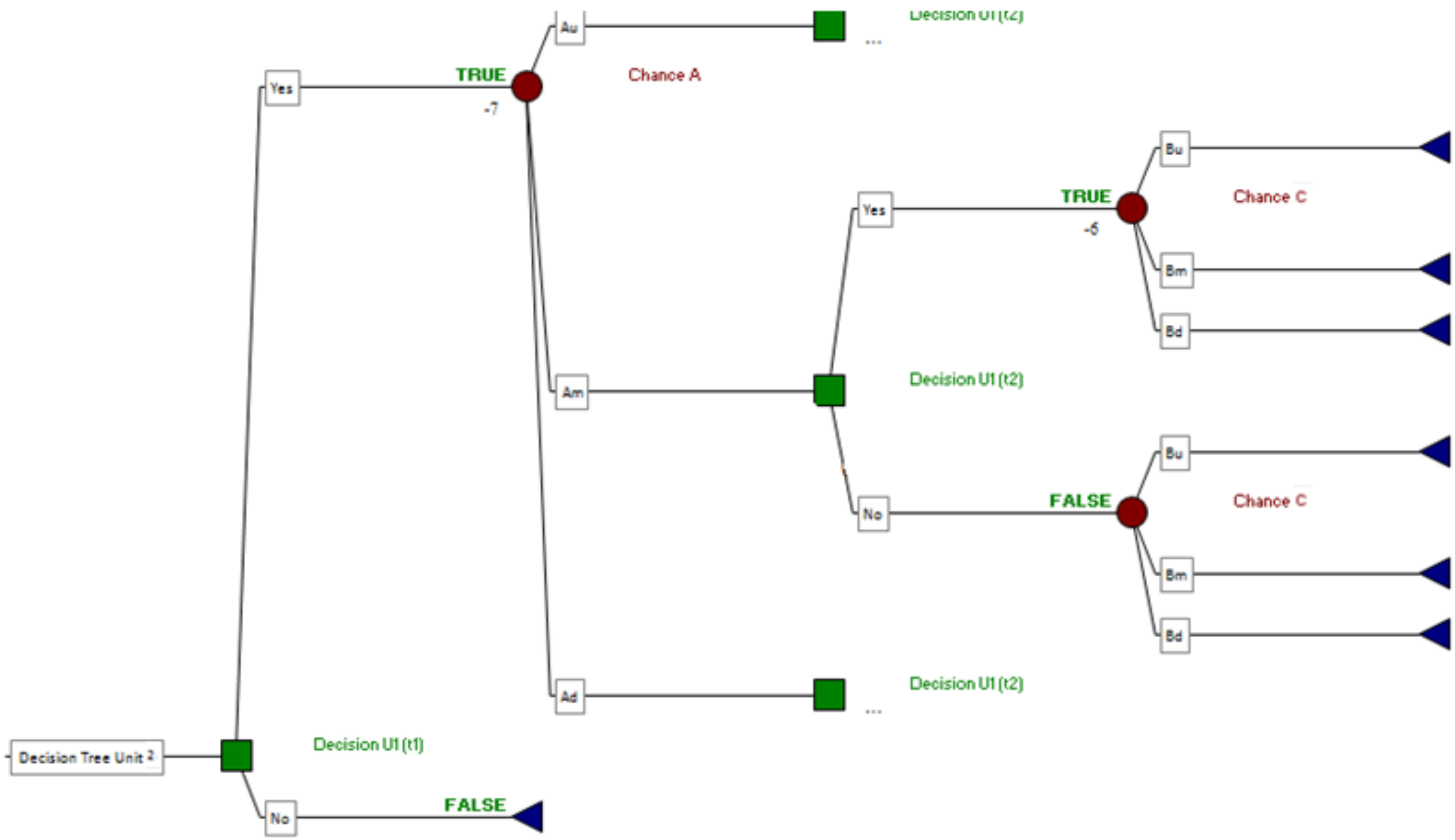
An Illustrative Example

- Decision making process for division 1



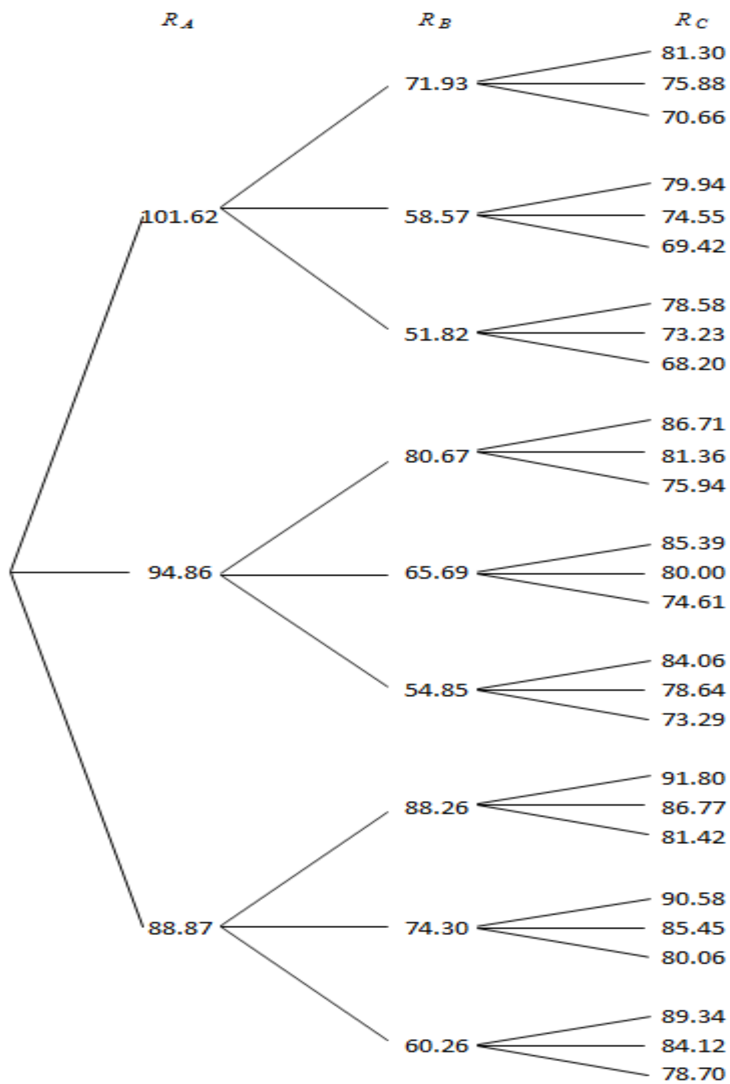
An Illustrative Example

- Decision making process for division 2



An Illustrative Example

- The corporate state of nature tree



An Illustrative Example

- Objective function
  - Using the mean-risk model, the CEO maximizes her certainty equivalent, *Maximize CE=*$[EV_T - \lambda * LSAD_T]$
  - Using lower semi-absolute deviation (LSAD) as the risk measure to focus on expected downside risk
- Model constraints
  - Logical consistency constraints
  - Capital resource constraints
  - Risk constraints
- Incorporating risk management strategies
  - Entails RM cost and alters cash flow/payoff structure
  - Modify mainly capital resource constraints

An Illustrative Example

- Logical consistency constraints

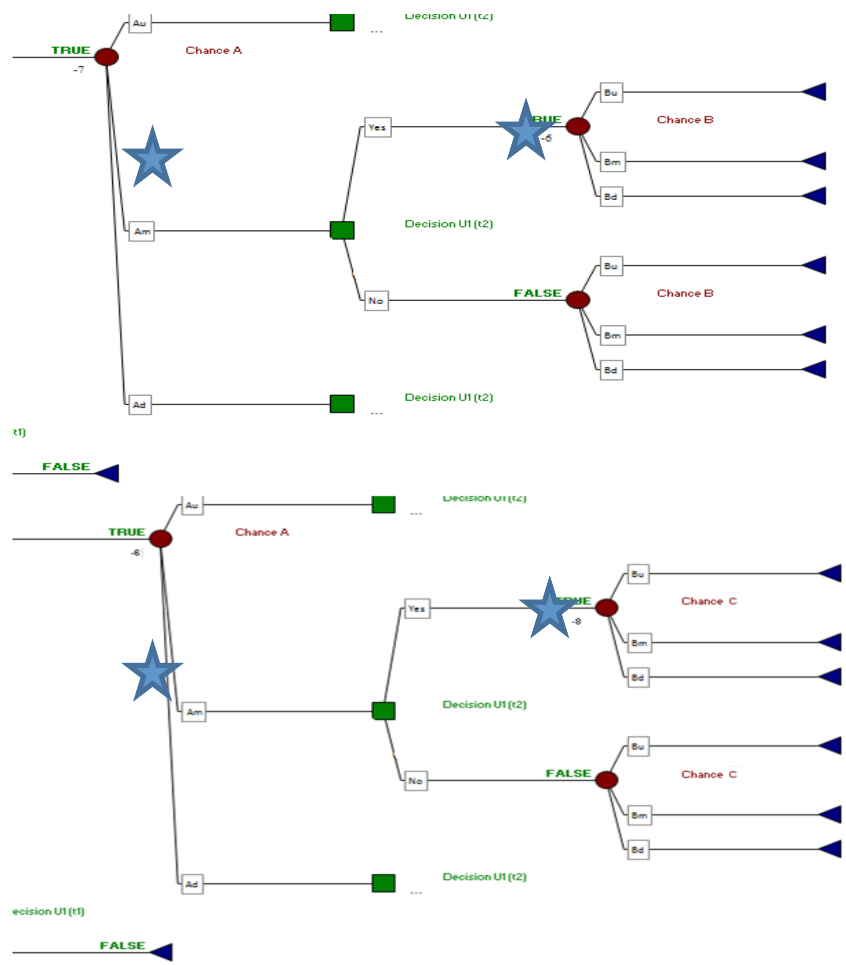
$X_{1,0,Y}+X_{1,0,N}=1$
$X_{2,0,Y}+X_{2,0,N}=1$
$X_{1,1,Y}+X_{1,1,N}=X_{1,0,Y}$
$X_{1,2,Y}+X_{1,2,N}=X_{1,0,Y}$
$X_{1,3,Y}+X_{1,3,N}=X_{1,0,Y}$
$X_{2,1,Y}+X_{2,1,N}=X_{2,0,Y}$
$X_{2,2,Y}+X_{2,2,N}=X_{2,0,Y}$
$X_{2,3,Y}+X_{2,3,N}=X_{2,0,Y}$

- Capital resource constraints

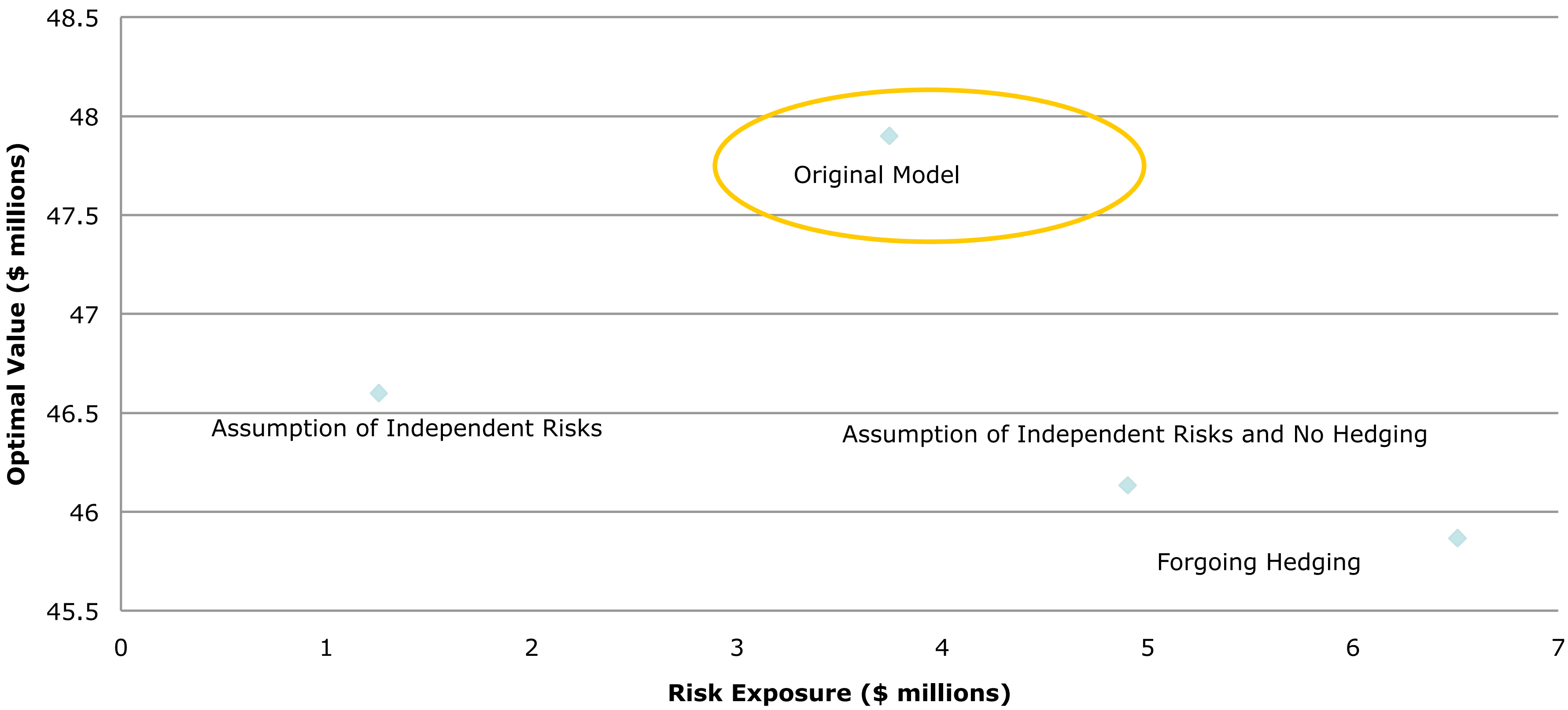
$-Inv_{1,0}*X_{1,0,Y}-Inv_{2,0}*X_{2,0,Y}+I_0-V_0=0$
$-Inv_{1,1}*X_{1,k,Y}-Inv_{2,1}*X_{2,k,Y}+(1+r_f)*V_0-V(k)=0$
$(A(k)-B(i,j))*X_{1,k,Y}+(A(k)-C(i,j))*X_{2,k,Y}+(1+r_f)*V(k)-V(i,j)=0$
All capital surplus $V>=0$, i.e., the company cannot run out of cash due to the budget constraint
$k=1, 2, 3$ denotes the states at $t=1$, $i=1,...,9$ and $j=1,\ldots,3$ for states at $t=2$

Optimal Decisions

Investment		Risk Management	
Panel A: Loan Division (Division 1)			
Period 1 (at $t = 0$)			
$X_{1,0,Y}$	1	$\alpha_{1,0}$	0.0197
$X_{1,0,N}$	0		
Period 2 (at $t = 1$)			
$X_{1,1,Y}$	1	$\alpha_{1,1}$	0
$X_{1,1,N}$	0	$\alpha_{1,2}$	0.6444
$X_{1,2,Y}$	1	$\alpha_{1,3}$	1
$X_{1,2,N}$	0		
$X_{1,3,Y}$	1		
$X_{1,3,N}$	0		
Panel B: Insurance Division (Division 2)			
Period 1 (at $t = 0$)			
$X_{2,0,Y}$	1	$\alpha_{2,0}$	0.7371
$X_{2,0,N}$	0		
Period 2 (at $t = 1$)			
$X_{2,1,Y}$	1	$\alpha_{2,1}$	0
$X_{2,1,N}$	0	$\alpha_{2,2}$	0.5924
$X_{2,2,Y}$	1	$\alpha_{2,3}$	1
$X_{2,2,N}$	0		
$X_{2,3,Y}$	1		
$X_{2,3,N}$	0		



Comparison of Optimal Values and Risk Measures



- Robustness results

Summary

- Capital Budgeting
  - Study optimal capital budgeting in a multi-divisional firm with dependent risk exposures
- Corporate risk management and ERM
  - Operationalize the concepts of ERM by incorporating risk management into the corporate decision making process
- Decision analysis
  - Modeling a managerially relevant problem of integrated corporate decision making
  - Providing and improving techniques
- Extensions and future research

Thank You!

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