# Regulation Risk

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# Overview

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We show that certain prudential rules might increase risk instead of lowering it.

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It is then important to investigate what are the real implications of these rules.

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We study a particular instance of this new type of risk. Our analysis is based on the dichotomy continuous/discontinuous for the movements of prices.

We show in this case how model risk and regulation risk combine into a market risk.

# Discontinuity of prices and consequences

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Movements responsible for market risk then come from two distinct origins: variance (or volatility) and jump intensity.

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Ignoring the fact that there are two independent dimensions entails a model risk.

We show that, under simplifying assumptions, this risk, combined with the VaR constraint imposed on financial firms, leads to a market risk.

# Stable motions

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Let us stress that:

1. we do not pretend that stable processes are the best models for price movements. More complex pure jump processes, such as for instance CGMY ones, are probably more adapted.
2. The discussion below remains valid with other infinite activity pure jump processes.

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$$\varphi(u) = \begin{cases} \exp\left\{i\mu u - \sigma^\alpha |u|^\alpha \left[1 - i\beta \operatorname{sign}(u) \tan\left(\frac{\alpha\pi}{2}\right)\right]\right\} & \text{if } \alpha \neq 1 \\ \exp\left\{i\mu u - \sigma |u| \left[1 + i\beta \operatorname{sign}(u) \frac{2}{\pi} \ln(u)\right]\right\} & \text{if } \alpha = 1 \end{cases}$$

# Stable motions

A stable motion is defined by four parameters:

1. $\alpha \in (0, 2]$. When $\alpha < 2$, it quantifies the distribution of the size of jumps: during a given period, and for all integer $j$, the mean number of jumps with size of the order of $2^j$ is proportional to $2^{-j\alpha}$. As a consequence, a large $\alpha$ corresponds to a small jump intensity, and vice versa.
2. $\sigma > 0$ is a scale parameter: if the process is multiplied by $a > 0$, then $\sigma$ turns to $a\sigma$. In the Gaussian case, *i.e.* $\alpha = 2$, the variance is equal to $2\sigma^2$. This means that $\sigma$ governs volatility.
3. $\mu$ is a location parameter: if one adds $a$ to the process, then $\mu$ becomes $\mu + a$.
4. $\beta$ ranges in $[-1, 1]$ and is a symmetry parameter. When $\beta = 0$, the distribution of increments is symmetric around $\mu$.

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There is no reason to believe that they remain constant in time. We thus consider local versions $\alpha(t)$ et $\sigma(t)$.

# An empirical study

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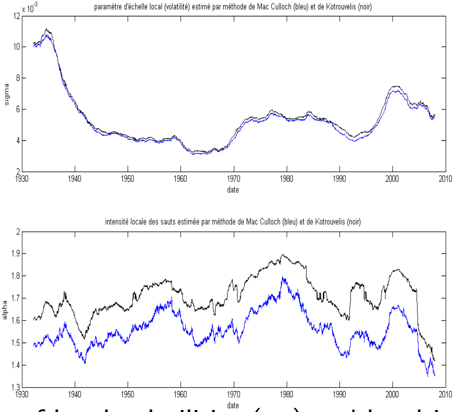
Each value of $\alpha$ and $\sigma$ is estimated using a centred moving window containing 2000 points.

# An empirical study




Compared evolution of local volatilities (up) and local jump intensities (bottom), with Mac Culloch and Kotrouvelis methods on daily quotes of the S&P 500 between 01/03/1928 and 02/01/2012. Values estimated with a centred moving window of 2000 points.

# An empirical study

Both methods yield very similar results for $\sigma$. Estimations of $\alpha$ are a little bit more different. However, both estimations in this case give almost parallel curves: this is sufficient for us, as our aim is to compare the evolutions of $\sigma$ and $\alpha$.

Since 1960 or so, jump intensity and volatility evolve in an opposite way: when $\sigma$ increases, the jump intensity decreases (since $\alpha$ increases) and vice versa: when the market is less "nervous", it is more prone to large jumps.

Evolution of recent years conforms that volatility has significantly decreased at the expanse of a notable increase of the local jump intensity.

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VaR (*Value at Risk*) at confidence level $1 - p$ and horizon $T$, which is the quantity such that the probability that losses at horizon $T$ are larger than VaR is $p$:

$$\mathbb{P}(X_T < -\mathrm{VaR}) = 1 - p.$$

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TCE (*Tail Conditional Expectation*) at confidence level $1-p$ and horizon $T$, which is defined as:

$$\mathrm{TCE} = \mathbb{E}\left(X_T \mid X_T < -\mathrm{VaR}\right).$$

# Consequence on risk measures

Under the assumption that prices follow a stable motion with $\beta = 0$, the asymptotic behaviour of VaR is given by:

$$\mathrm{VaR} \simeq \sigma \left( \frac{C_\alpha}{2(1-p)} \right)^{\frac{1}{\alpha}}, \quad \text{where} \quad C_\alpha = \frac{1-\alpha}{\Gamma(2-\alpha)\cos(\pi\alpha/2)}.$$

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This implies that VaR increases linearly with volatility. One can show that it also decreases when $\alpha$ increases. This does correspond to intuition : a larger jump intensity translates into a larger VaR, and thus a more risky market.

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As a consequence, in a situation where $\sigma$ decreases while the jump intensity increases (that is, $\alpha$ decreases), which is what we have observed empirically, then, under a constant VaR, TCE will increase.

For instance, if $\alpha$ moves from 1.75 to 1.4 (as measured on the S&P 500), then, if VaR remains constant, TCE is multiplied by 1.5. This means that a constraint on the VaR has a negative impact.

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The model that is implicit in prudential regulation reduces variations to the sole volatility parameter, while a more adequate model should also consider the independent contribution of jumps.

Regulation risk consists in imposing a VaR constraint: because jumps are ignored, keeping VaR constant increases TCE, and thus market risk.