Advancements in Common Shock modeling

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Tokio Marine Technologies LLC

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- *Jimmy Su* PhD & SVP, *Tokio Marine Technologies*

Background:

Relevant literature:

- **Advanced Correlations** (2012 MetaRisk® Conference), *Steve White*
- **The Common Shock Model** (Variance Vol. 1/Issue 1 1997) *Glenn Meyers*
- **The Calculation of Aggregate Loss Distribution from Claim Severity and Claim Count distributions** (PCAS, LXX, 1983), *Philip Heckman*, *Glenn Meyers*

Background:

Common Shock modeling (*a.k.a. Contagion modeling*):

- Attempts to account for the additional, systematic, uncertainty within Insurance data:
  - **Claim Counts (Frequency) distributions:**
    - Exposure-base, of the insured, changes over-time.
      - Specifically: over the range of historical data.
    - IBNR claims must be estimated.
    - External drivers can cause change in claim frequencies:
      - Severe recession → increase fire claims
  - **Claim Size (Severity) distributions:**
    - External drivers of severities:
      - Inflation
      - Underwriting cycle
      - Macroeconomic factors

Basic Common Shock/Contagion model

Frequency & **Severity Common Shock/Contagion:**

Frequency

Let: N_i, for $i = 1, 2\ \cdots, K$ be K claim count $RV's$, from K lines of business:

$$N_1|C \sim FreqDist(\boldsymbol{\theta}_1|C)$$
$$N_2|C \sim FreqDist(\boldsymbol{\theta}_2|C)$$
$$\vdots$$
$$N_K|C \sim FreqDist(\boldsymbol{\theta}_K|C)$$

Where:
- $\boldsymbol{\theta}_i$ = vector of distribution parameters
- $C \sim Dist(\ E[C] = 1, Var[C] = c\ )$

Where c is a *scalar-valued* parameter, the "*Frequency Contagion parameter*".

Severity

Let: X_k be the loss size *R.V.*, given a claim, from the k^{th} line of business:

$$X_k \sim\ Dist_k\left(E[X_k] = \mu_k,\ \ Var[X_k] = \sigma_{x_k}^2\right)$$

Let: $\beta \sim Dist(E(\beta) = 1, Var(\beta) = b)$

Where b is a *scalar-valued* parameter, called the *Severity Contagion parameter.*

Multiply each X_k by the *same* realization of β: $\beta X_k \qquad k = 1, 2\ \cdots, K$

Poisson Frequency

Since the same *Frequency Contagion RV*, C, is used within each N_i:

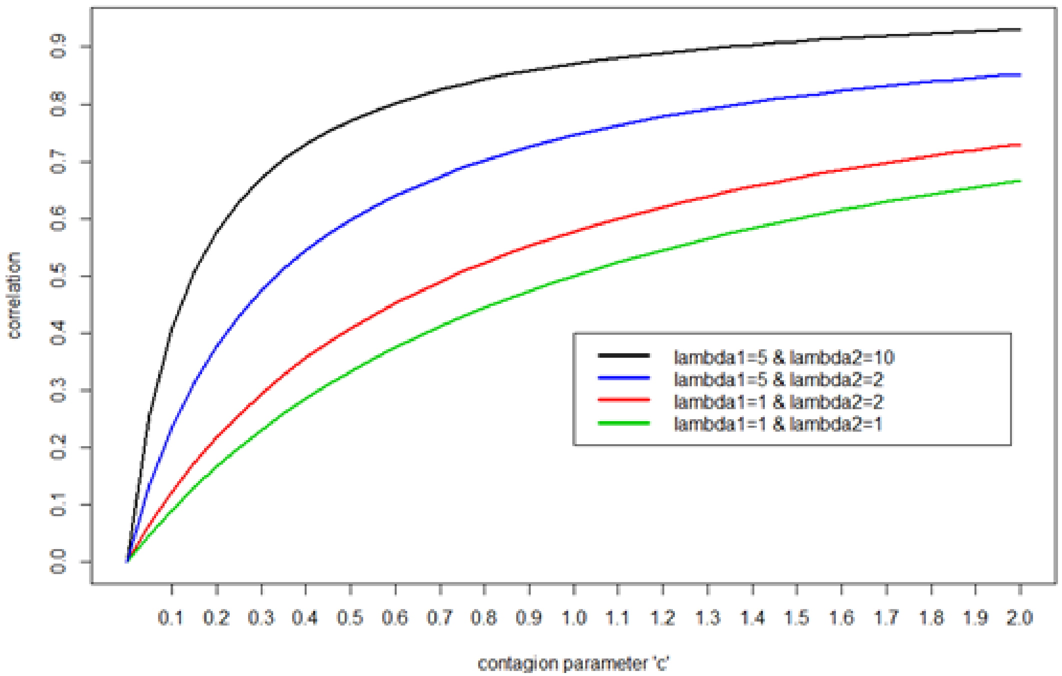
- N_i, for $i = 1, 2 \cdots, K$ are *correlated*:

If $N_i \sim \boldsymbol{Poisson}(\lambda_i)$, then $define$: $N_i | \boldsymbol{C} \sim Poisson(\boldsymbol{C}\lambda_i)$ where $\boldsymbol{C} \sim Dist(\, E[C] = 1, Var[C] = c\,)$

Then, for $1 \leq i, j \leq K$, and $i \neq j$, the correlation between N_i, N_j is:

$$\rho_{N_i,N_j} = \sqrt{\frac{c\lambda_i}{1 + c\lambda_i}} \sqrt{\frac{c\lambda_j}{1 + c\lambda_j}}$$

- As $c \to 0 \implies \rho_{N_i,N_j} \to 0$
  - weak, or absent, contagious environment
- As $c \to \infty \implies \rho_{N_i,N_j} \to 1$
  - a strong contagious environment



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Negative Binomial Frequency

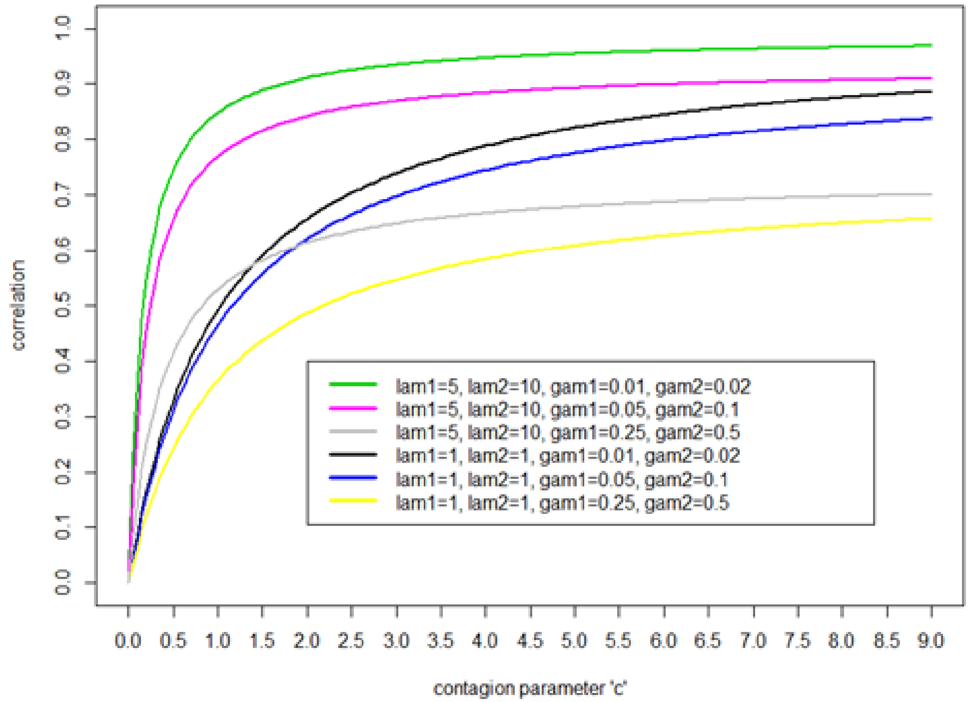
If $N_i \sim \boldsymbol{NegBinomial}(\lambda_i,\ \gamma_i)$ then define $N_i|\boldsymbol{C} \sim NegBin(\boldsymbol{C}\lambda_i,\ \gamma_i)$

where: $C \sim Dist(\,E[C] = 1, Var[C] = c\,)$ and $(\lambda_i = \text{mean} \quad \gamma_i = \text{dispersion parmeter})$

For $1 \le i,j \le K$, and $i \ne j$, the correlation between N_i, N_j is:

$$\rho_{N_i,N_j} = \sqrt{\frac{c\lambda_i}{1+\lambda_i(c+c\gamma_i+\gamma_i)}}\sqrt{\frac{c\lambda_j}{1+\lambda_j(c+c\gamma_j+\gamma_j)}}$$

- As $c \to 0 \implies \rho_{N_i,N_j} \to 0$
  - weak, or absent, contagious environment
- As $c \to \infty \implies \rho_{N_i,N_j} \to \sqrt{\frac{1}{1+\gamma_1}}\sqrt{\frac{1}{1+\gamma_2}}$
  - *Strong* contagious environment



Binomial Frequency

- Denote the best-fitting Binomial distributions to each of the K lines, by:
  - $\widehat{N}_i \sim \boldsymbol{Bin}(\hat{n}_i, \hat{p}_i)$ for $1 \leq i \leq K$
- Let: $\hat{p}^* = \max(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_n)$

To simulate Claim counts from the line:

- Adjust the parameter, p, of each distribution, by the constant ratio: $\hat{p}_i/\hat{p}^*$

$$N_i|p \sim Bin\left(\hat{n}_i, \left(\frac{\hat{p}_i}{\hat{p}^*}\right)p\right) \quad \text{where} \quad p \sim Beta\left(\alpha = \frac{1}{c},\ \beta = \frac{1}{c}\left(\frac{1-\hat{p}^*}{\hat{p}^*}\right)\right)$$

Hence:

- $E(N_i) = E_p\left[E_{N_i}(N_i \mid p)\right] = \hat{n}_i \cdot \hat{p}_i$ (the mean of the best-fitting Binomial distribution, to that line)

$$Var(N) = \frac{\hat{n}_i\hat{p}_i(1-\hat{p}_i) + c\hat{p}^*\left[\hat{n}_i\hat{p}_i\left(1-\frac{\hat{p}_i}{\hat{p}^*}\right) + (\hat{n}_i)^2(\hat{p}_i)^2\left(\frac{1}{\hat{p}^*}-1\right)\right]}{1+c\hat{p}^*}$$

Binomial Frequency: Variance

$$Var(N) = \frac{\hat{n}_i\hat{p}_i(1-\hat{p}_i) + c\hat{p}^*\left[\hat{n}_i\hat{p}_i\left(1-\frac{\hat{p}_i}{\hat{p}^*}\right) + (\hat{n}_i)^2(\hat{p}_i)^2\left(\frac{1}{\hat{p}^*}-1\right)\right]}{1+c\hat{p}^*}$$

Several observations can be made from this expression:

- $c \to 0 \implies Var(N_i) \to \hat{n}_i\hat{p}_i(1-\hat{p}_i)$ (variance of *best-fitting* Binomial, to business line i)
- $Var(N_i)$ is an *increasing* function of c.
- $c \to \infty \implies Var(N_i) \to \hat{n}_i\hat{p}_i\left(1-\frac{\hat{p}_i}{\hat{p}^*}\right) + (\hat{n}_i)^2(\hat{p}_i)^2\left(\frac{1}{\hat{p}^*}-1\right)$

Note:

- If $\hat{p}_i = \hat{p}^*$, for line i:
- $c \to \infty \implies Var(N_i) \to \hat{n}_i\hat{p}_i(0) + (\hat{n}_i)^2(\hat{p}_i)^2\left(\frac{1}{\hat{p}_i}-1\right) = (\hat{n}_i)^2\hat{p}_i(1-\hat{p}_i) > \hat{n}_i\hat{p}_i(1-\hat{p}_i)$

Binomial Frequency: Correlation

For $1 \leq i, j \leq K$, and $i \neq j$:

$$\rho_{N_1,N_2} = \frac{\hat{n}_i \hat{p}_i \hat{n}_j \hat{p}_j \left(\frac{c(1-\hat{p}^*)}{1+c\hat{p}^*}\right)}{\sigma_i \sigma_j}$$

Where:

$$\sigma_i = \sqrt{\frac{\hat{n}_i \hat{p}_i (1-\hat{p}_i) + c\hat{p}^* \left[\hat{n}_i \hat{p}_i \left(1 - \frac{\hat{p}_i}{\hat{p}^*}\right) + (\hat{n}_i)^2 (\hat{p}_i)^2 \left(\frac{1}{\hat{p}^*} - 1\right)\right]}{1+c\hat{p}^*}}$$

Observations:

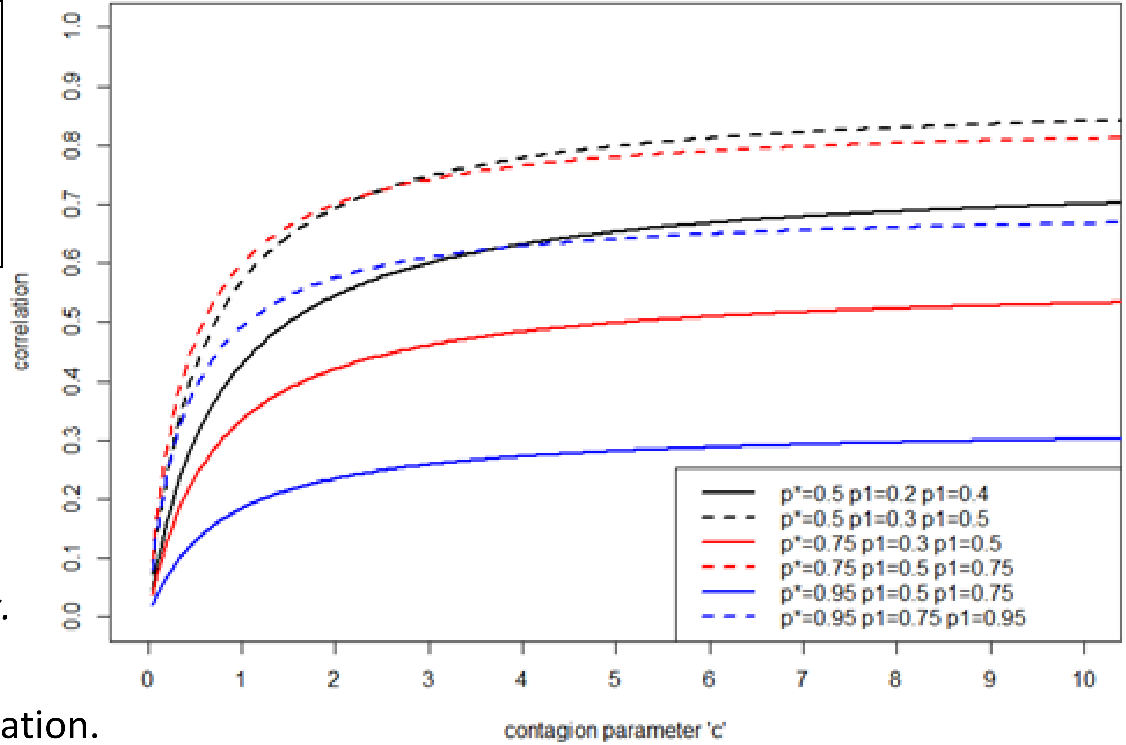
- $c \to 0 \implies \rho_{N_1,N_2} \to 0.$
- ρ_{N_1,N_2} in an *increasing* function of c.
- $c \to \infty \implies \rho_{N_i,N_j} \to \dfrac{1}{\sqrt{\left(1+\frac{(\hat{p}^* - \hat{p}_i)}{\hat{n}_i \hat{p}_i (1-\hat{p}^*)}\right)\left(1+\frac{(\hat{p}^* - \hat{p}_j)}{\hat{n}_j \hat{p}_j (1-\hat{p}^*)}\right)}}$

Binomial: Correlation vs. c (per level of Binomial parameters, and p^*)

Recall:

As: $c \to \infty \implies$

$$\rho_{N_i,N_j} \to \frac{1}{\sqrt{\left(1+\frac{(\hat{p}^*-\hat{p}_i)}{\hat{n}_i\hat{p}_i(1-\hat{p}^*)}\right)\left(1+\frac{(\hat{p}^*-\hat{p}_j)}{\hat{n}_j\hat{p}_j(1-\hat{p}^*)}\right)}}$$



Observations:

- For a given value of $\hat{p}^* < 1$:
  $\hat{p}_i$ and $\hat{p}_j$ larger $\implies$
  $\implies$ ***higher*** correlation.
- For fixed values of $\hat{p}_i$ and $\hat{p}_j$, *s.t.* $\min\{\hat{p}_i, \hat{p}_j\} < \hat{p}^*$:
  - higher $\hat{p}^* \implies$ ***lower*** correlation.
- As $c \to \infty$:
  - If $\hat{p}_i = \hat{p}_j = \hat{p}^* < 1 \implies \rho_{N_i,N_j} \to 1$
  - If $\min\{\hat{p}_i, \hat{p}_j\} < \hat{p}^*$, then $\hat{p}^* \to 1 \implies \rho_{N_i,N_j} \to 0$

Summary: Frequency Contagion

- The introduction of Contagion ***preserves*** *the* ***mean*** of the frequency distribution:
  - $E(N_i) = E_C\left[E_{N_i}(N_i \mid C)\right] = \lambda_{\boldsymbol{i}}$ for $N_{\boldsymbol{i}} \sim \boldsymbol{Poisson}(\lambda_{\boldsymbol{i}})$ or $N_{\boldsymbol{i}} \sim \boldsymbol{Neg}Binomial(\lambda_i,\ \gamma_i)$

- However, the ***variance*** of the claim count *RV,* will be increased:
  - $Var(N_i) = Var_C\left[E[N_i|C]\right] + E_C\left[Var[N_i|C]\right] = \lambda_i(1 + c \cdot \lambda_i)$ $\quad$ $(\ N_i \sim Poisson(\lambda_i)\ )$
  - $Var(N) = \lambda\big(1 + \lambda(c + c\gamma + \gamma)\big)$ $\quad$ $(\ N_i \sim NegBinomial(\lambda_i,\ \gamma_i)\ )$
  - $Var(N) = \frac{\hat{n}_i\hat{p}_i(1-\hat{p}_i)+c\hat{p}^*\left[\hat{n}_i\hat{p}_i\left(1-\frac{\hat{p}_i}{\hat{p}^*}\right)+(\hat{n}_i)^2(\hat{p}_i)^2\left(\frac{1}{\hat{p}^*}-1\right)\right]}{1+c\hat{p}^*}$ $\quad$ $(\ \hat{N}_i \sim Bin(\hat{n}_i, \hat{p}_i)\ )$

- The “frequency contagion *RV*”, C, can follow *any* distributional form.
  - The only restrictions are:
    1. The distribution must have ***positive support***
    2. The mean must be 1: $E[C] = 1$

Summary: Frequency Contagion, cont……

- The correlation only depends on: (Regardless of which frequency distribution is used)
  1. *Parameters* of the Frequency distribution, and
  2. "*Contagion parameter*", c.

Ex: Poisson: $\rho_{N_i,N_j} = \sqrt{\frac{c\lambda_i}{1+c\lambda_i}}\sqrt{\frac{c\lambda_j}{1+c\lambda_j}}$

- Even though the same contagion parameter, c, is used, *across all lines-of-business*:
  - ρ_{N_i,N_j} will NOT equal ρ_{N_m,N_n}, **unless** $(\lambda_i, \lambda_j) = (\lambda_m, \lambda_n)$
    - Hence, ***only one parameter***, c, will induce a whole, non-constant, correlation matrix.
    - The induced correlation matrix will be *"automatically" determined,* by:
      - The marginal distribution, of each line.
      - The value of the *contagion parameter*, c.

Proposed Aggregate Common-Shock/ Contagion model

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Proposed Aggregate Contagion model

Let: $FreqDist(\boldsymbol{\theta}_k|C)$ be the *best-fitting* distribution to the claim count data, for the k^{th}-line:

- $N_k^*|C \sim FreqDist(\boldsymbol{\theta}_k|C)$ where $C \sim Dist(\, E[C] = 1, Var[C] = c\,)$

Let $Dist_{X_k}(\mu, \sigma_{X_k}^2)$ be the *best-fitting* distribution to the loss size data , for the k^{th}-line:

Assume that X_k can be *decomposed* into the product of two RV; $\boldsymbol{\beta Z_k}$, s.t.:

$$\boxed{Var(X_k) \approx Var[\beta Z_k]}$$

- $X_k \sim Dist_k\big(E[X_k] = \mu_k,\ Var[X_k] = \sigma_{X_k}^2\big)$ is the observed, empirical, claim size data.
- β is the *severity contagion* RV, s.t. $\beta \sim Dist(E(\beta) = 1, Var(\beta) = b)$
    - β represents the ***systematic*** component of the losses process X_k.
- Z_k the *underlying* loss RV.
    - Z_k represents the ***idiosyncratic*** component of the losses process X_k.
        - *i.e.* the true, underlying, loss process.

Aggregate Contagion model - requirements

Requirements for the proposed Aggregate contagion model:

1. β be *independent* of Z_k, and
2. $E(Z_k) = \mu_{z_k} = \mu_k = E(X_k)$
3. The distribution of β must have ***positive support***
4. The mean must be 1: $E[\beta] = 1$

These conditions ensure that:

- $E[\beta Z_k] = E[\beta]E[Z_k] = 1 \cdot \mu_{z_k} = \mu_k = E(Z_k)$
- $Var[\beta Z_k] = \sigma_{z_k}^2 + b\left(\mu_k^2 + \sigma_{z_k}^2\right)$

In summary, the *Severity component* of the proposed Aggregate Contagion model is s.t.:

$\boldsymbol{\beta Z_k}$ where: $\begin{cases} Z_k \sim RanDist_k\left(E[Z_k] = \mu_k,\ \ Var[Z_k] = \sigma_{z_k}^2\right) \\ \beta \sim RanDist(E(\beta) = 1,\ \ Var(\beta) = b),\ \text{with } b \geq 0 \end{cases}$

- such that: $E(Z_k) = \mu_{z_k} = \mu_k = E(X_k)$ and $Var(Z_k) = \sigma_{z_k}^2 \leq \sigma_{x_k}^2 = Var(X_k)$

Aggregate Contagion model - description

The proposed severity contagion model is consistent with the common shock/ contagion modeling paradigm, since:

- In the absence of a contagious environment, it should be inferred that; $\sigma_{Z_k}^2 \approx \sigma_{X_k}^2$, and hence:
  - $\sigma_{X_k}^2 = Var(X_k) \approx \ Var[\beta Z_k] = \sigma_{Z_k}^2 + b\left(\mu_k^2 + \sigma_{Z_k}^2\right)$, which implies that; $b \approx 0$.
- Conversely, in the presence of a strong contagious environment, it should be inferred that:
  - $\sigma_{Z_k}^2 \ll \sigma_{X_k}^2$, which, by the same argument, implies that $Var[\beta] \gg 0$, or $b \gg 0$.

Conversely, under the same assumption that: $Var(X_k) \approx Var[\beta Z_k]$, we have that:

- $b \approx 0$ implies that $\sigma_{Z_k}^2 = \ Var[\beta Z_k] \approx Var(X_k) = \sigma_{X_k}^2$, which implies a weak contagious environment, and:
- $b \gg 0 \implies \sigma_{X_k}^2 = Var(X_k) \approx \ Var[\beta Z_k] = \sigma_{Z_k}^2 + b\left(\mu_k^2 + \sigma_{Z_k}^2\right) \gg \sigma_{Z_k}^2 \implies$ strong contagious environment.

Calibration proposed Contagion model

The proposed represents a refinement to the *current* Severity contagion approach, in the literature: $\beta \boldsymbol{X_k}$.

- By fitting $\boldsymbol{X_k}$ to the empirical data, and then multiplying by β:
  - $Var[\beta X_k] = \sigma_{x_k}^2 + b\left(\mu_k^2 + \sigma_{x_k}^2\right) > \sigma_{x_k}^2 \approx Var[data]$
    - ***i.e.***: $\beta \boldsymbol{X_k}$ will ***over-estimate*** the variance of the observed, per-claim, loss size data.

However, the proposed aggregate contagion model will only be useful if:

1. $\mathrm{Var}(X_k)$ can faithfully be modeled by the product of β, and Z_k:
   - ***i.e.***: $Var(X_k) \approx Var[\beta Z_k]$
2. A *calibration scheme* exist, which is able to effectively isolate the contribution of β, and Z_k, to the variation of the data.

We investigate these assumptions in the following case studies.

Case Studies

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Case Study #1: Property Natural Peril Severity Data

We first investigate the assumptions for a *single line-of-business*:

Data: 670 claims between years 2003 and 2012

- Property Natural Peril (severe convective storm)
- For a single company.
- Occurrence basis.
- Losses over 10-years: 2003 – 2012
- Loss sizes are in units of $1,000.

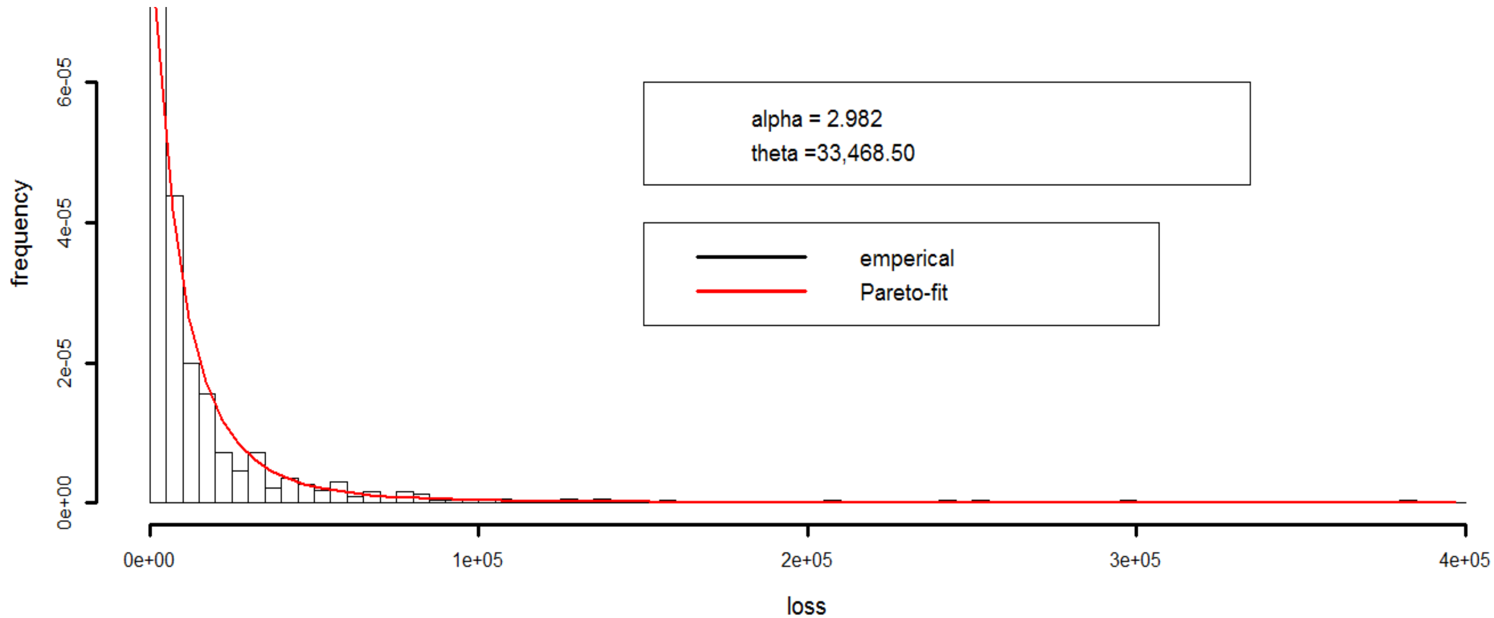
Frequency calibration:

- We use a Poisson distribution for the claim counts.
- The parameter value λ is set equal to the empirical average annual claim counts:
  - $\lambda = \bar{X} = 67.$
  - Solve: $Var(N) = \lambda(1 + c \cdot \lambda)$ for c:
    - $c = \frac{\widehat{Var}(N)}{\lambda^2} - \frac{1}{\lambda} = \approx 0.115$
      - $\widehat{Var}(N) = sample\ variance$ and $\lambda = \bar{X} = 67$

Case Study #1: Property Natural Peril Severity Data

Severity calibration:

- Using the per-claim severity data, over the full 10-years:
  - The best-fitting distribution to the per-claim severity, $\boldsymbol{X}$, is:
    - ***Pareto** distribution,* with
      - $\alpha_{MLE} = \textit{shape parameter} = 2.982$, and
      - $\theta_{MLE} = \textit{scale parameter} = 33{,}468$



Case Study #1: Property Natural Peril Severity Data

But ….. we need the distribution of the *pure*, *underlying*, loss process; ***Z***

Let:

$$S^* = \sum_{i=1}^{N^*} \beta Z_i$$

$$\Longrightarrow \quad Var[S^*] = Var[Z_i]E(\,N^*) + (E[Z_i])^2\; Var(\,N^*) + b \cdot \{Var[Z_i]E(\,N^*) + (E[Z_i])^2 E(N^{*2})\}$$

Note: This equation incorporates both frequency and severity contagion, and does not depend on the distributional form of the claim count RV (Poisson, Negative Binomial, or Binomial).

And:

- $E(\,N^*) = \lambda$
- $Var(\,N^*) = \lambda(1 + c \cdot \lambda)$

Hence:

$$Var[S^*] = \sigma_z^2\lambda + \mu_z^2\lambda(1 + c\lambda) + b \cdot \{\sigma_z^2\lambda + \mu_z^2[\lambda(1 + c\lambda) + \lambda^2)]\} = \lambda(1 + b)(\mu_z^2 + \sigma_z^2) + \lambda^2\mu_z^2[b + c + bc]$$

$$\Longrightarrow \quad b = \frac{Var[S^*] - \lambda\sigma_z^2 - \lambda b(\mu_z^2 + \sigma_z^2) - \lambda\mu_z^2 - \lambda^2\mu_z^2 c}{\lambda^2\mu_z^2(1 + c)}$$

Case Study #1: Property Natural Peril Severity Data

Calibration of the *pure*, *underlying*, loss process; $\boldsymbol{Z}$

Set: $Var[S^*] = \widehat{Var}[S^*] =$ *empirical* variance of the ***annual aggregate losses***:

$$\Rightarrow \quad b = \frac{\widehat{Var}[S^*] - \lambda\sigma_z^2 - \lambda b(\mu_z^2 + \sigma_z^2) - \lambda\mu_z^2 - \lambda^2\mu_z^2 c}{\lambda^2\mu_z^2(1+c)}$$

And use the following values (all based on the sample):

- The mean of the Poisson frequency distribution $(\lambda = 67)$
- The modeled mean of the per-claim severity distribution $(\mu_z = 17{,}842)$
- The modeled variance of the per-claim severity $(Var[\beta Z] = \sigma_x^2 = 32{,}329^2)$
- The *empirical* variance of the annual aggregate losses $(Var[S^*] = 697{,}245^2)$
- The frequency contagion parameter $(c = 0.115\ )$

$$\Rightarrow \qquad b = \frac{Var[S^*] - \lambda \cdot Var[\beta Z] - \lambda\mu_z^2 - \lambda^2\mu_z^2 c}{\lambda^2\mu_z^2(1+c)} \quad \approx \quad \mathbf{0.13}$$

And:

$$\Rightarrow \qquad \sigma_{\boldsymbol{z}}^2 = \frac{Var[\beta Z] - b\mu_z^2}{1+b} = 29{,}634^2$$

Case Study #1: Property Natural Peril Severity Data

Finally, using:

- $\mu_z = 17{,}842$
- $\sigma_{\boldsymbol{z}}^2 = 29{,}634^2$

The distribution of the *pure*, *underlying*, loss process; $\boldsymbol{Z} \sim Pareto(\alpha_z, \theta_z)$ are determined to be:

- $\alpha_z = 3.137$, and
- $\theta_z = 38{,}133$

Now, we perform a simulation study of the Aggregate Annual *Layered* Losses

$$S^* = \sum_{i=1}^{N^*} \beta Z_i$$

Using the parameter values calibrated from the actual, empirical, data:

- $\boldsymbol{N}|\boldsymbol{C} \sim Poisson(\boldsymbol{C} \cdot \mathbf{67})$ and $\boldsymbol{C} \sim Gamma(\, E[C] = 1, Var[C] = \mathbf{0.115}\,)$
- $Z_i \sim Pareto(\mathbf{3.137},\ \mathbf{38{,}133}\,)$ and $\beta \sim Gamma(E(\beta) = 1,\ Var(\beta) = \mathbf{0.13})$

Case Study #1: Property Natural Peril Severity Data

We use the following Layers, of the *AAL*:

$$\begin{cases} Layer1: \ \mathbf{0-7.5M} \\ Layer2: \boldsymbol{7.5M - 20M} \\ Layer3: \boldsymbol{20M - 45M} \\ Layer4: \boldsymbol{45M - 70M} \\ Layer5: \boldsymbol{70M - 100M} \\ Layer6: \boldsymbol{100M - 200M} \end{cases}$$

Simulation procedure:

1. For each iteration of the simulation, generate 10 years of Aggregate Annual Losses under both:
   - The Traditional method
   - The Contagion method.
2. For each of the pre-defined Loss Layers (above) calculate the ***Annual Aggregate losses*** within each layer.
3. For each layer, calculate the CV of the Aggregate Annual losses, over the 10-years.

Repeat 100,000 times.

At this point, we have $100{,}000$ 10-year CV estimates, ***for each layer***

Case Study #1: Property Natural Peril Severity Data

We use the following Layers, of the *AAL*:

$$
\begin{cases}
Layer1: \ \mathbf{0-7.5M} \\
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\end{cases}
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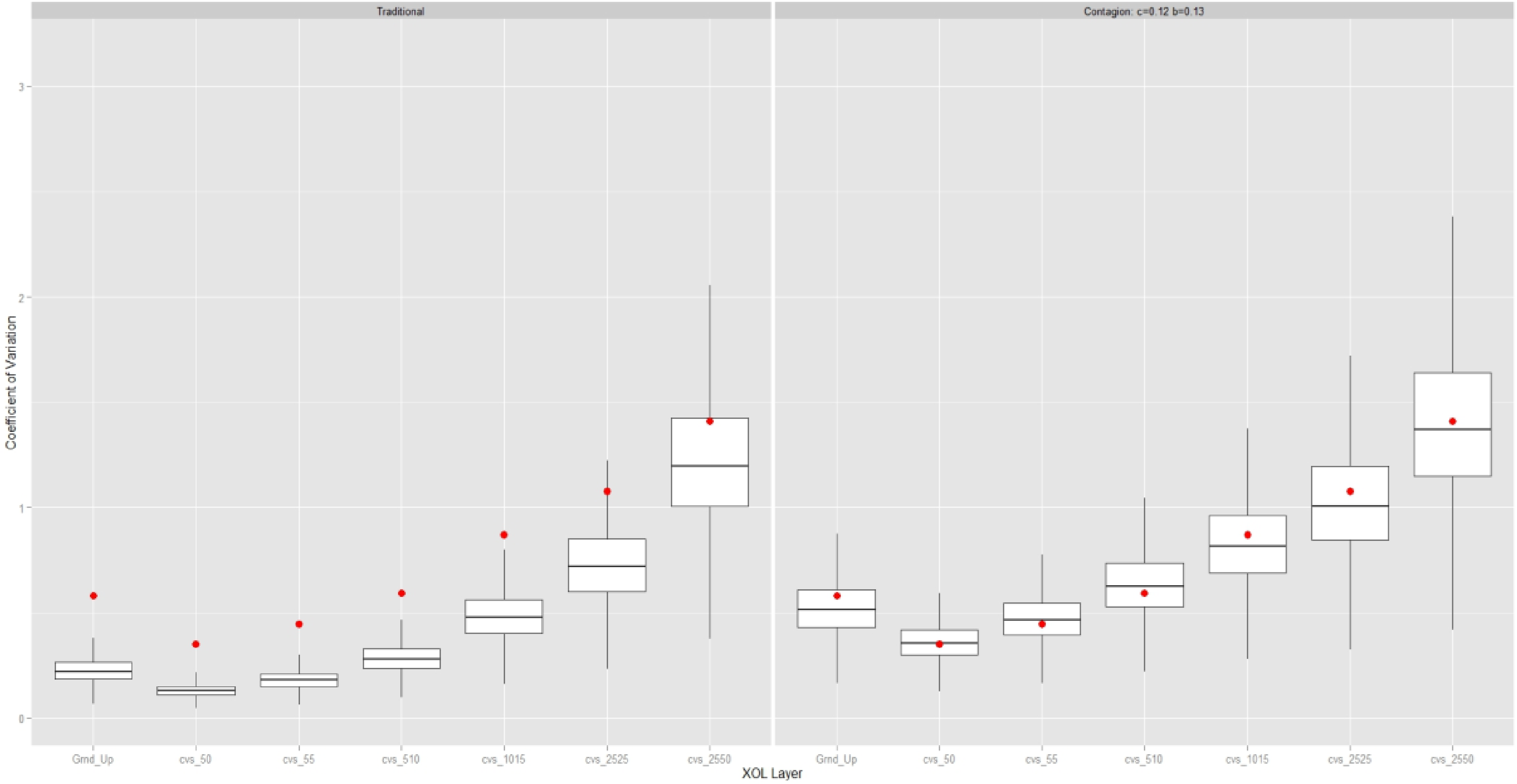
Case Study #1: Results

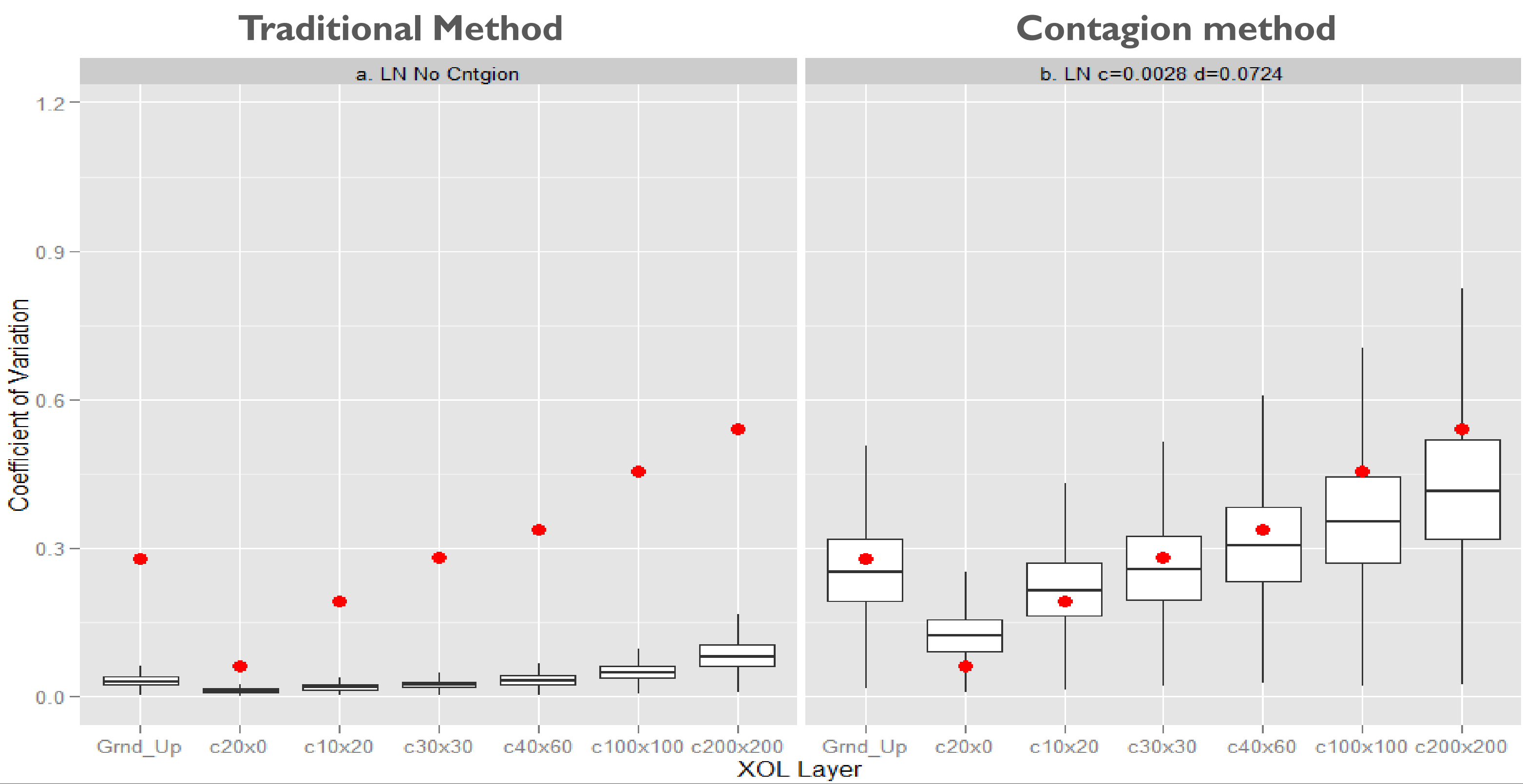
Traditional Collective Risk model | ***Proposed* Aggregate Contagion method**



Case Study #2: XYZ Insurance *GL* Claims Data

Provided loss data on:

- LOB: GL claims on transaction-level
- Losses over 5-years: 2009 – 2013
- Occurrence basis, Losses recorded after policy-limits and deductibles



Case Study: Summary

- **Traditional Collective Risk modeling**
  - No Frequency Correlation available
  - No Severity Correlation available within the LOBs
  - Produces *flawed* estimates of the Variation of Aggregate Annual claims.
    - Underestimates the Variation.

- **Traditional Collective Risk *underestimates*:**
  - Volatility of aggregate losses.
  - Volatility of aggregate losses within XOL layers.
  - Risk measure in terms of Spectral risk, TVaR or VaR.

- **Effect of Traditional Collective Risk Model:**
  - May *underestimate* Capital Requirements.
  - May *underprice* reinsurance contracts.

Contagion Model: Summary

- **Contagion Model**
  - Induce frequency correlation using frequency contagion
  - Induce severity correlation through severity contagion
- **Easy to understand through implied correlation and volatility**
- **Easy to implement within high performance simulation**
- **Represent the state-of-the-art in correlation treatment (*Meta Risk, ReMetrica*)**
- **Have demonstrated that contagion exists using real life data.**
- **Have showed that the contagion can better estimate:**
  - Volatility of aggregate losses
  - Risk measure in terms of Spectral risk, TVaR or VaR
  - Expected loss of XOL layers