Canadian Pensioners Mortality at Extreme Ages with Data as at December 31, 2012

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Outline

1. Introduction
2. Objectives & Questions
3. Differences at Older Ages
4. Extreme Ages Modeling
5. Materiality: q_x, e_x
6. Conclusion

1.1 Introduction

- Larger project: “Canadian Pensioners Mortality Trends by Region, Income and Cohort as at December 31st, 2012”
- Split larger project in smaller sections
- Work in Progress: data validation, measurement of level and trends, extrapolation,…
- Focus of this talk on part of this larger project: mortality at extreme ages => ending the tables

1.2 Acknowledgements

- Joint work with Étienne Vanasse, undergraduate student, with contribution from other students
- Funding
  - Chaire d'actuariat, Laval University
- Data and support
  - Canada Pension Plan, Office of the Chief Actuary, Ottawa (CPP)
  - Quebec Pension Plan, Régie des rentes du Québec, Québec (QPP)

1.3 Previous results: source

- Previous results on Canadian Pensioners Mortality CPM) published by the Canadian Institute of Actuaries (CIA) in 2013: Mortality Level and Trends
- See CIA website:
  - (http://www.cia-ica.ca/publications) 213003e.pdf, 213003t.pdf, 213012.pdf
- Shown differences in probabilities of death and mortality improvement rates by measurable variables
- Recent work: shown heterogeneity of mortality trends by region and income level for Canadian Pensioners, based on 2008-12-31 data

1.4 Previous results: variables

- Results: Deaths, Exposure, q_x , Improvement Rates IR(x) , tests, …
- Available by measurement variables:
  - Data source: QPP, CPP, CAN (QPP+CPP)
  - Age: 60 to 115 years
  - Gender: Male, Female
  - Income Class: 1 to 5 (*)
  - Period or years: 2005-2007 3-year was most recent period (1970 to 2007)
- * Income Class:

  1=low income (< 35% Maximum Pension),

  2=mid income (35% to 95%),

  3=high income (>95%), 4=2+3, 5=All

1.5 Previous results: extreme ages

- Less variation at extreme ages for data source, income, calendar year
- Previous work used Coale & Kisker (C&K) model to end mortality table: see section IV and Appendix J of CPM document on CIA website
- Model calibrated using all exposure and deaths from 1992 to 2007, Combined results for both data sources, all income
- Terminal value q_{115} : **0.54** for males, **0.51** females
- Gompertz model until age 95, using data specific to a particular combination of "source/gender/income/calendar period"
- Connection to C&K model from specific value at age 95 to common value at age 115
- q_{115} to q_{119} : constant, then 1.0 at 120

1.6 Messages from current work

- Reliable Canadian data at extreme ages
- Gompertz ?: not over age 100
- Flattening of Mortality ?: yes
- Disparities by Income ?: not over age 90
- Disparities by Region ?: not over age 90
  Disparities by Gender ?: yes
- Disparities by Calendar year ?:
  - not over age 98-100
- Which model over age 100 ?: C&K / Logistic
  - C & K retained

2.1 Objectives

- Update knowledge on mortality levels and trends: use most recent data (2012-12-31)
- Connect available data at extreme ages with a suitable model for later ages, using this specific information
- Determine which model is a better fit **with this specific set of data**

2.2 Questions

- How different is mortality by calendar year at older ages?
- What model is a better fit with this specific set of data:
  - C&K
  - Gompertz
  - Logistic: Kannisto, Beard
  - Denuit & Goderniaux and variant
- What is the best guess for mortality over age 105?
- How to connect with specific experience at lower ages?
- How material are the differences?

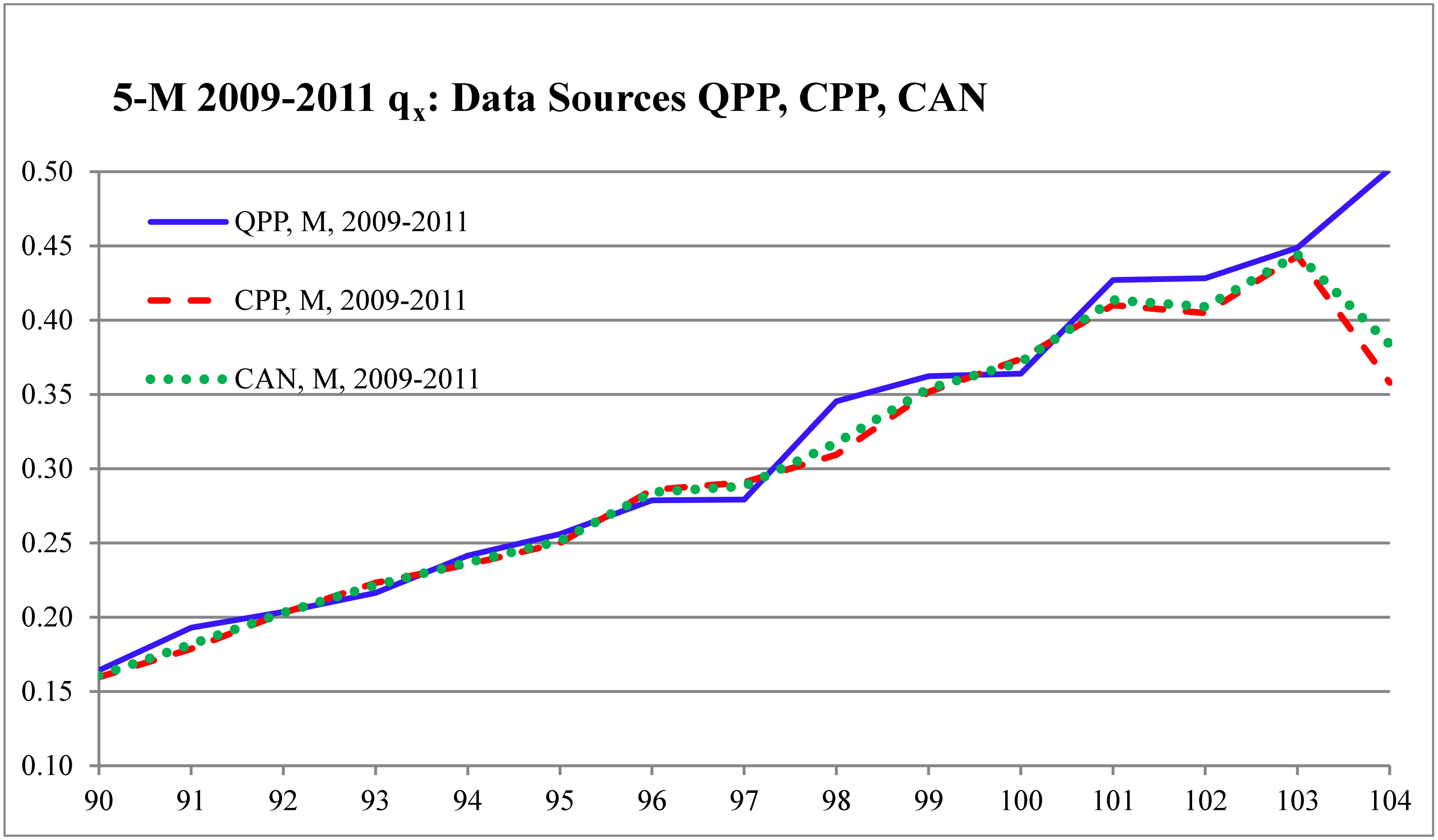
2.3 Data info

- **Recent**: from 1967 to **2012** (but use **2011** as end year)
- **Canadian** data source: appropriate to measure Canadian pensioners mortality
- **Complete and reliable**: All pensions paid according to CPP or QPP
- **Administrative**: not census or survey
- **Individual**: dates, pension paid
- **Limitations**: Retirees only, no survivors, less age specific info in earlier years, no info under age 60, no info by income over YMPE
- Proxy for mortality of Canadian private plans members

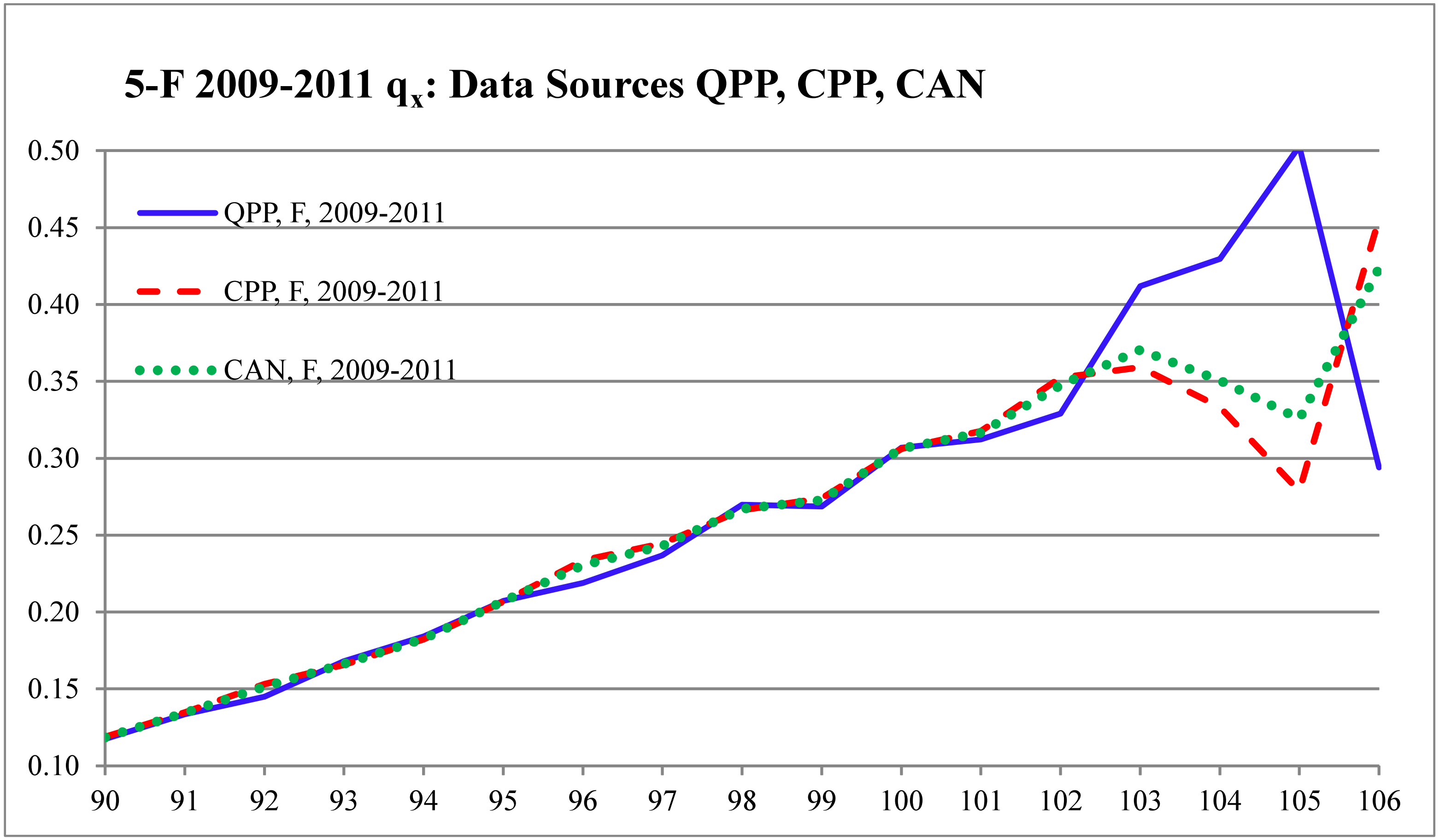
3.1 Differences at Older Ages

- Data over age 90: Illustration of differences in q_x
- Relevance of Gender: yes, until age 101
- Relevance of Data Source: no (age 90-91)
- Relevance of Income Class: no (age 91 Females)
- Relevance of Calendar Year: it depends…
    - 2009-2011 vs. 1986-2011: triennial vs. consolidated experience for all years

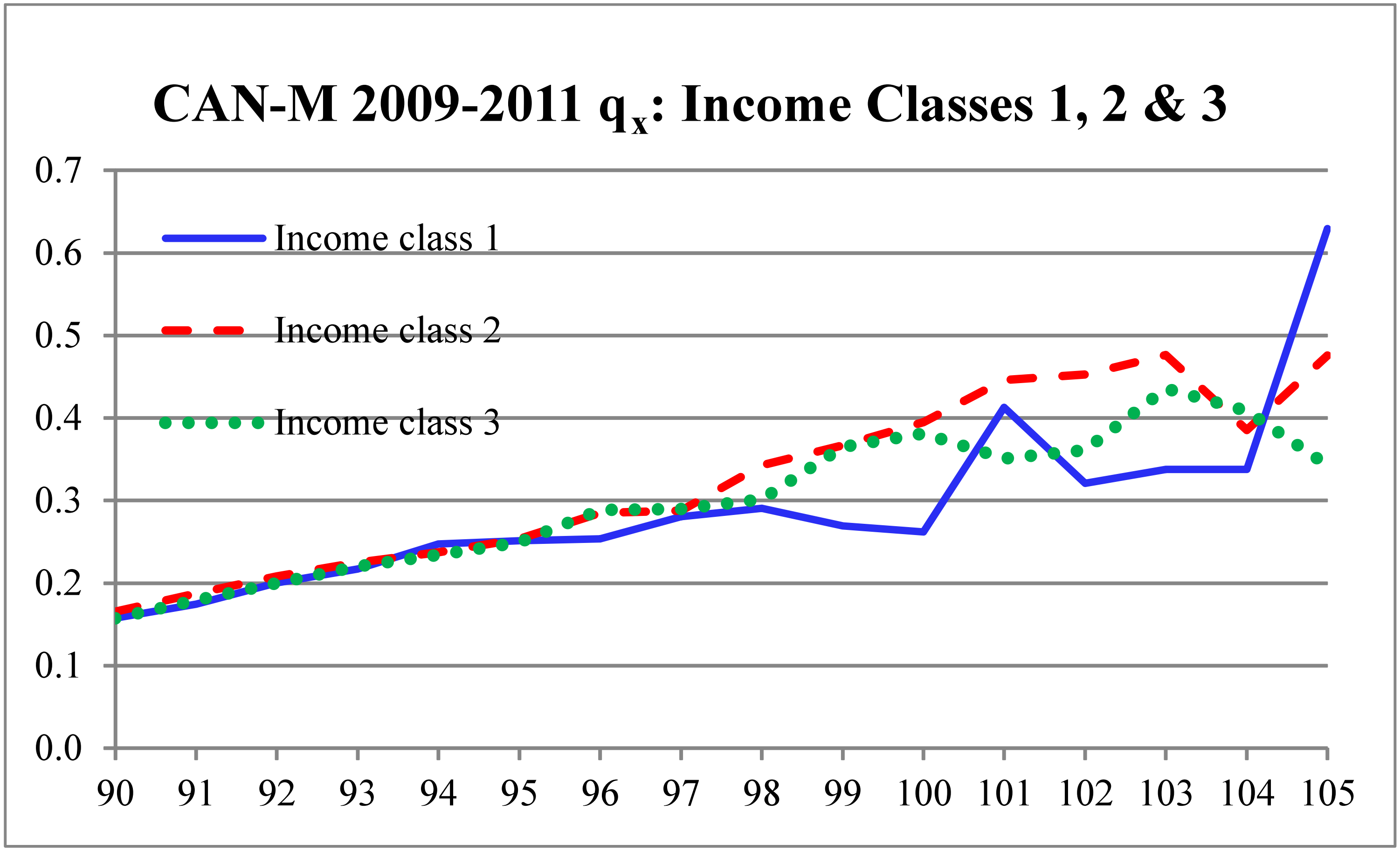
3.2 Differences: Data Source, Males



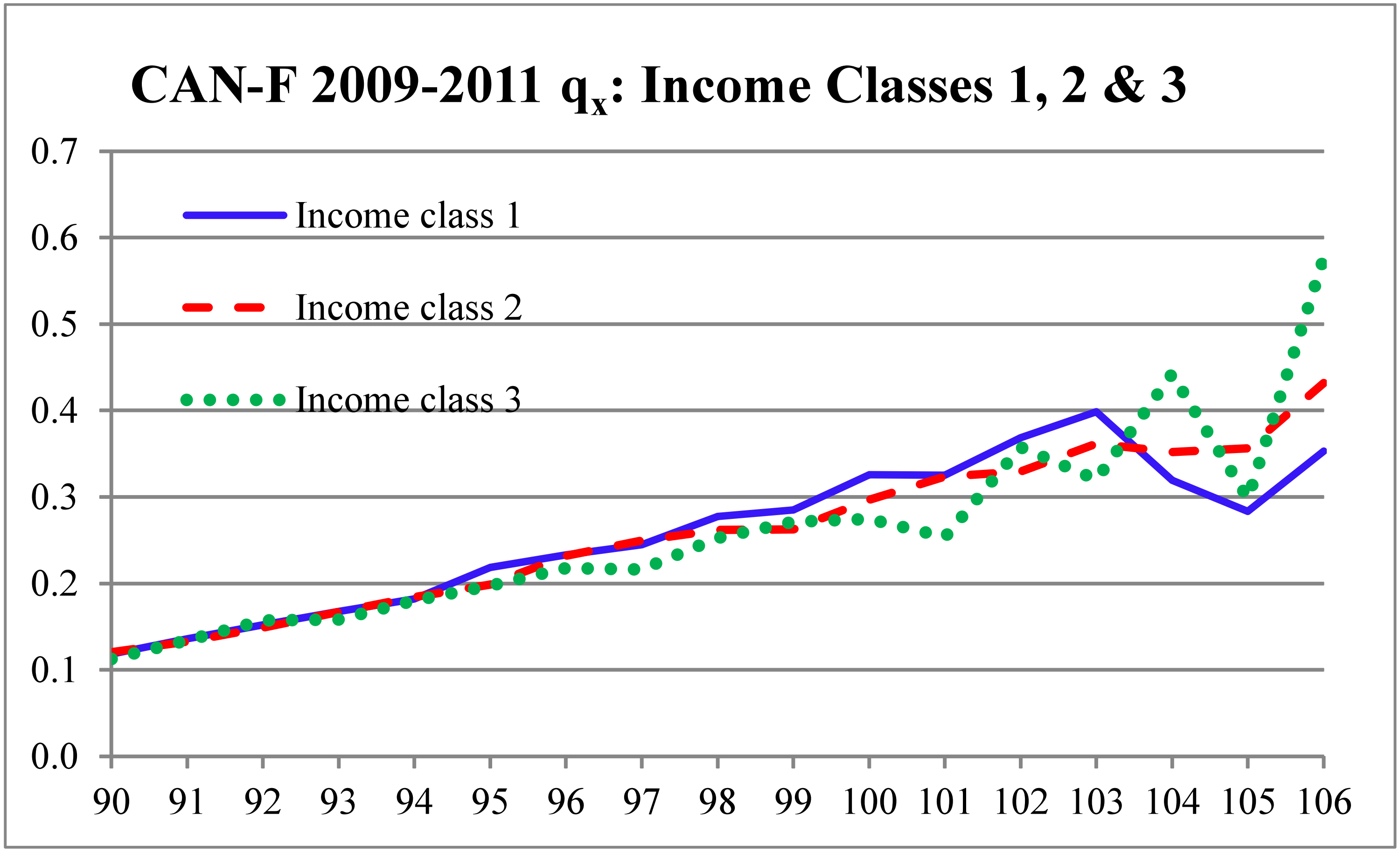
3.3 Differences: Data Source, Females



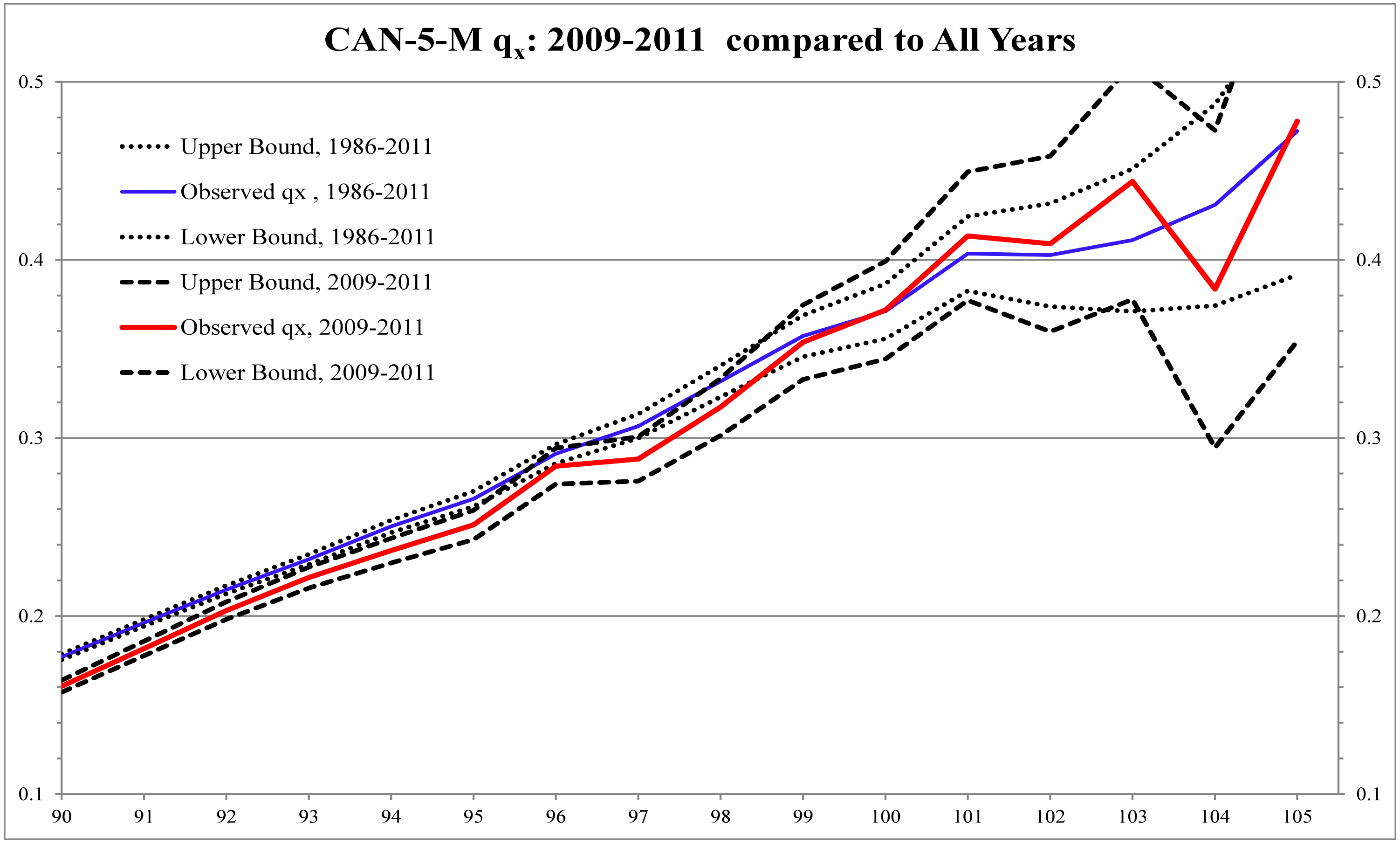
3.4 Differences: Income, Males



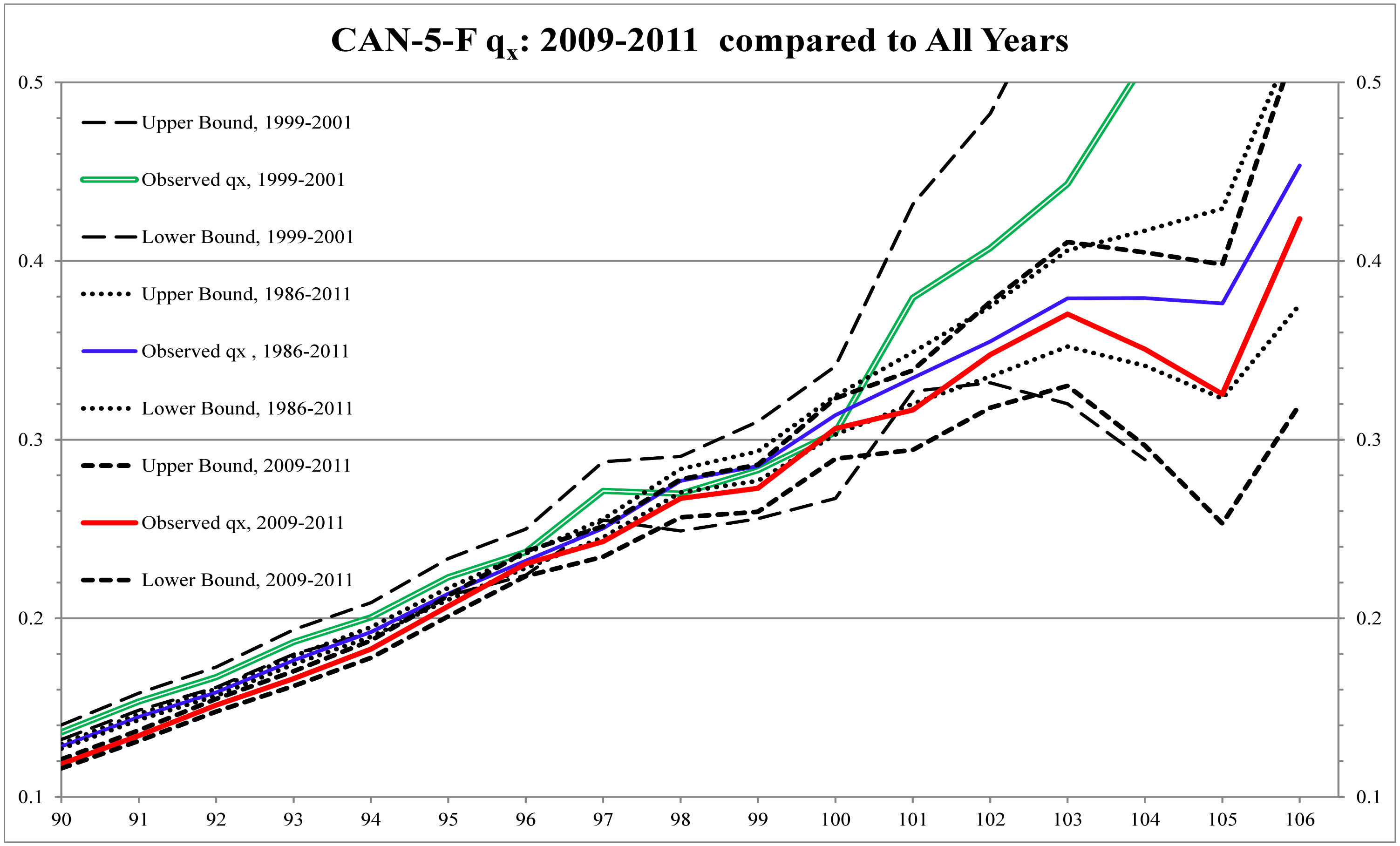
3.5 Differences: Income, Females



3.6 Differences: Calendar Year, Males



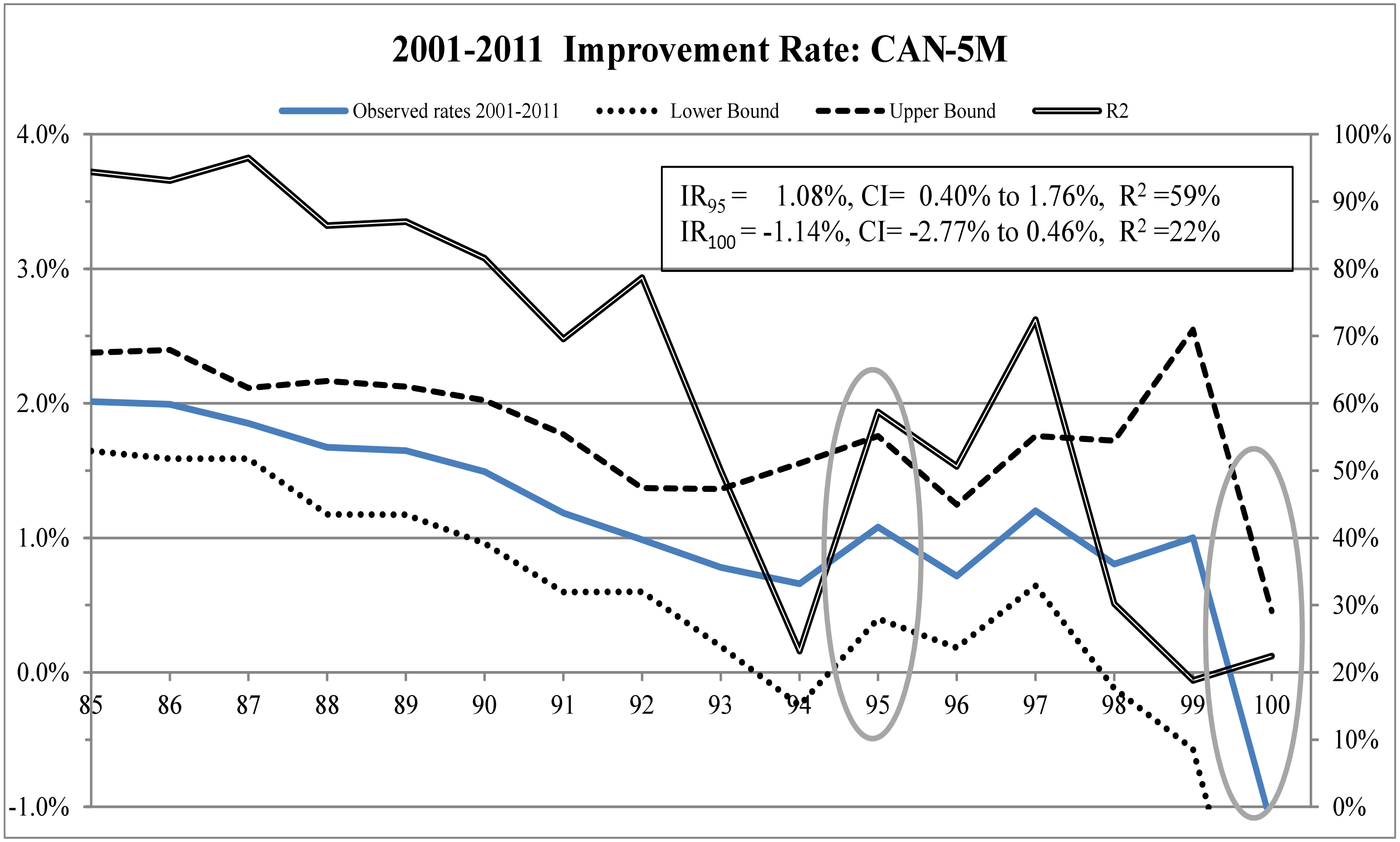
3.7 Differences: Calendar Year, Females



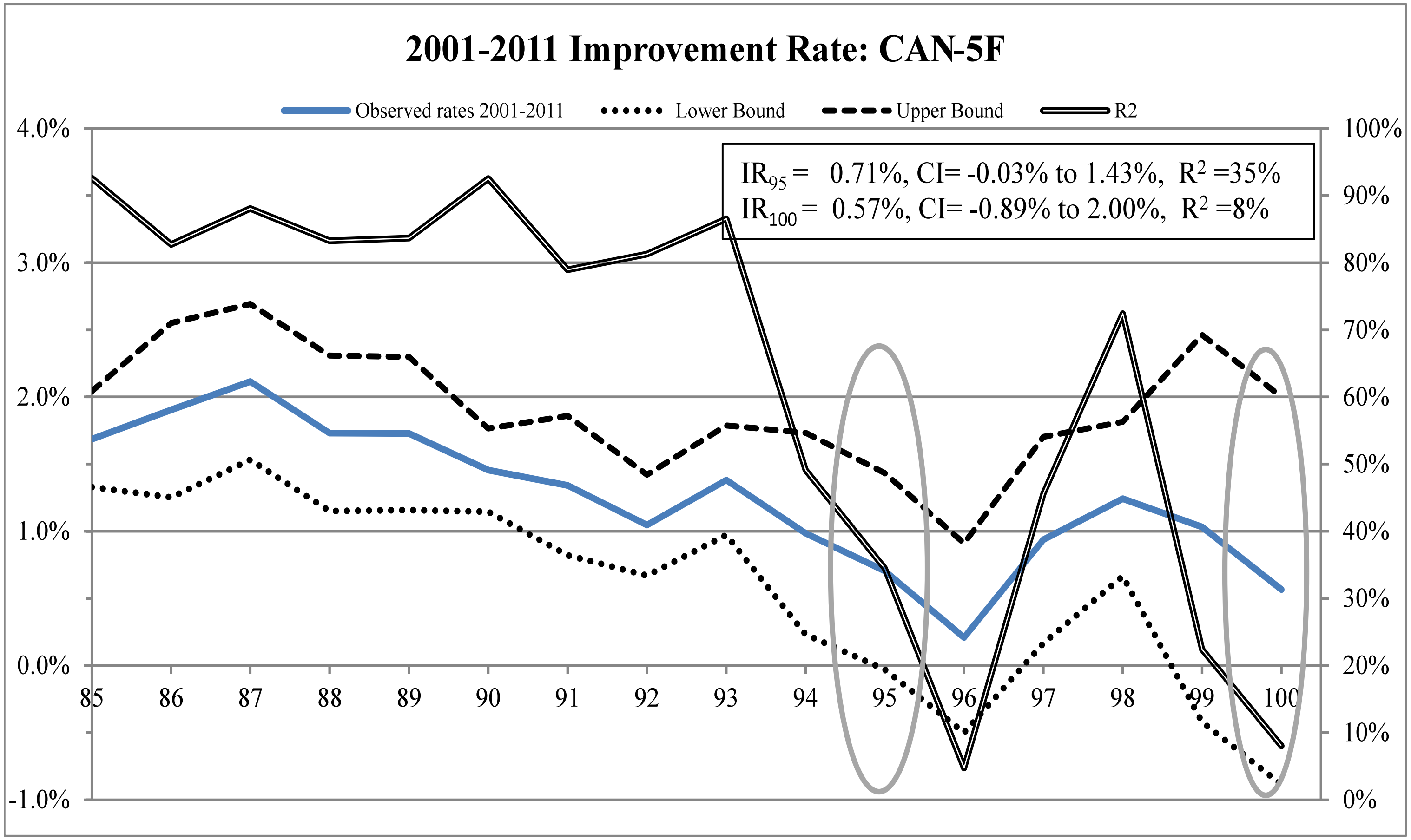
3.8 Improvement Rates: Older Ages

- Results from Weighted Linear Regression for CAN-5-M and CAN-5-F
- Ages: 85 to 100
- Time period: 2001-2011: 10-year regression
- Show adequacy of assumption:
  - q_x decreases with time at age 90
  - No correlation with time at age 100
- Decreasing trend of IR(x) with age x
- Increased uncertainty with higher ages:
  - lower R^2
  - larger 95% Confidence interval
  - negative improvement rate at higher ages

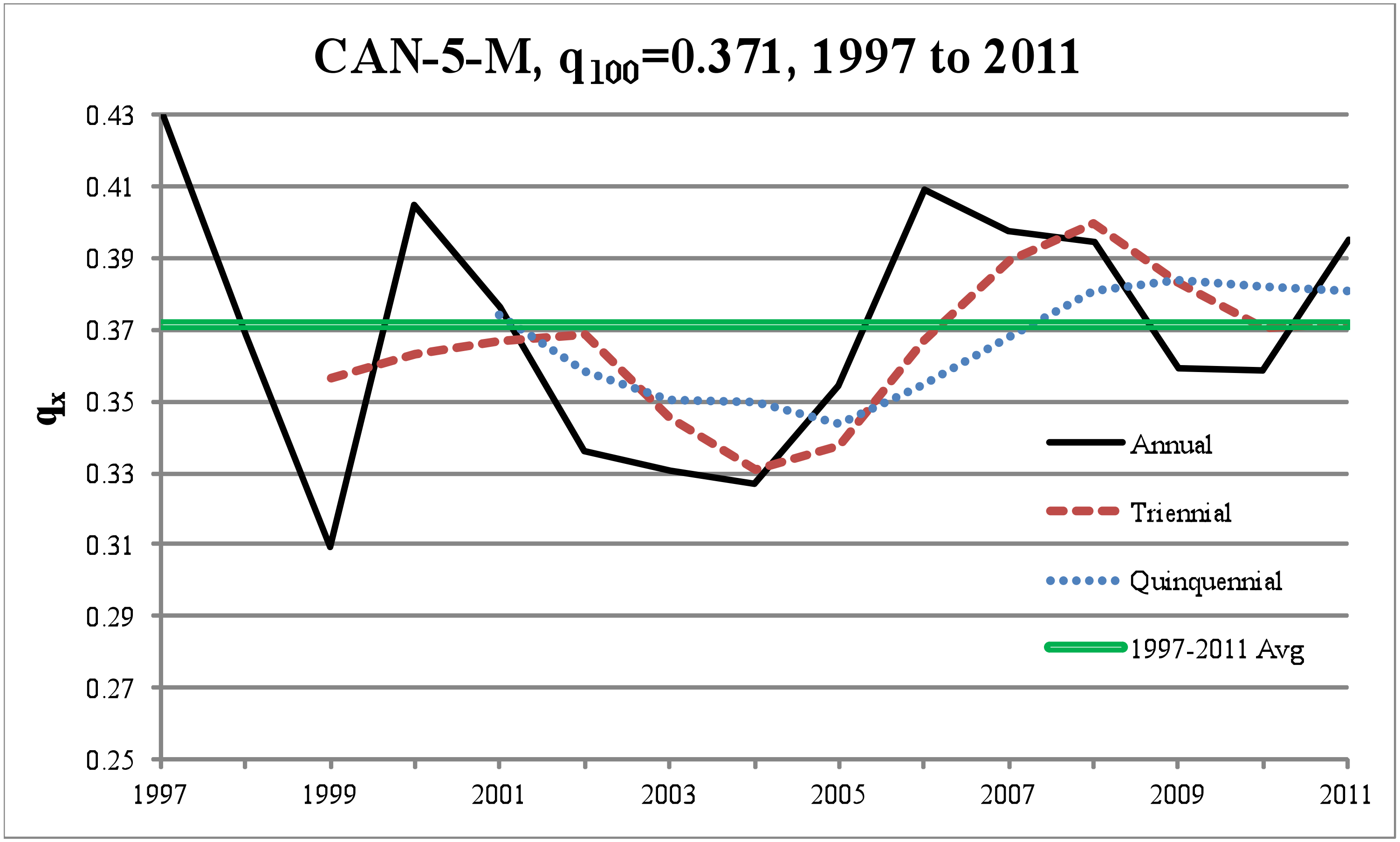
3.9 Improvement Rates: Males



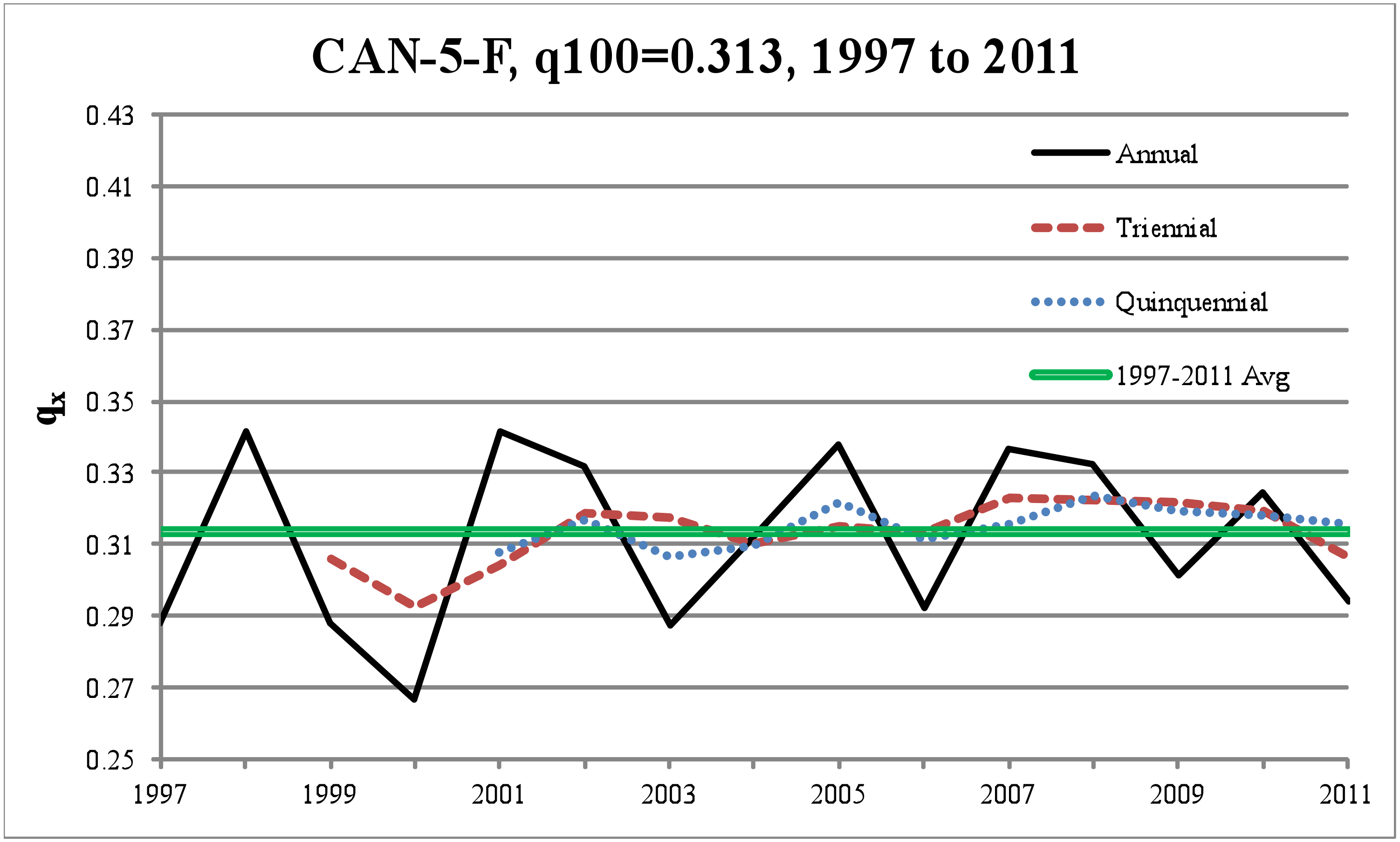
3.10 Improvement Rates: Females



3.11 CAN-5-M at age 100: Males



3.12 CAN-5-F at age 100: Females



4.1 Methodology

- Formulas, illustrations for various methods
  - Gompertz
  - Beard
  - Kannisto
  - Coale & Kisker (C&K)
  - Denuit & Goderniaux (D&G)
  - D & G variant (an adaptation)
- Charts from age 90 to age 110: Observed q_x, 95% Confidence Interval bounds, model value
- Data:
  - CAN-5 2009-2011 tables (M and F), age <100
  - Consolidated 1996-2011 for ages ≥ 100

4.2 Formulas

- Gompertz: linear increase of $\ln(I_x)$

$$I_x = Bc^x; \quad \ln(I_x) = \ln(B) + x\ln(c)$$

- Beard: logistic, with asymptote of I_x to N_2/N_4

$$I_x = \frac{N_2 \exp(N_3 x)}{1 + N_4 \exp(N_3 x)}$$

- Kannistö: logistic, with asymptote of I_x to 1

$$I_x = \frac{N_2 \exp(N_3 x)}{1 + N_2 \exp(N_3 x)}$$

4.2 Formulas (cont'd)

- Coale & Kisker: linearly decreasing addition to $\ln(\hat{\mu}_x)$

  estimate last age $\hat{\psi}$ at which no further addition to $\ln(\hat{\mu}_x)$

  $\hat{\mu}_{\hat{\psi}}$ at age $\hat{\psi}$; $\ln(\mu_{\Phi}) = \ln(\hat{\mu}_{90}) + \sum_{i=91}^{\Phi} k_i$; $k_{\Phi} = 0$

- Denuit & Goderniaux (D&G): quadratic function of q_x

  with $q_{130} = 1.0$; $\left.\frac{\partial q_x}{\partial x}\right|_{x=130} = 0$;

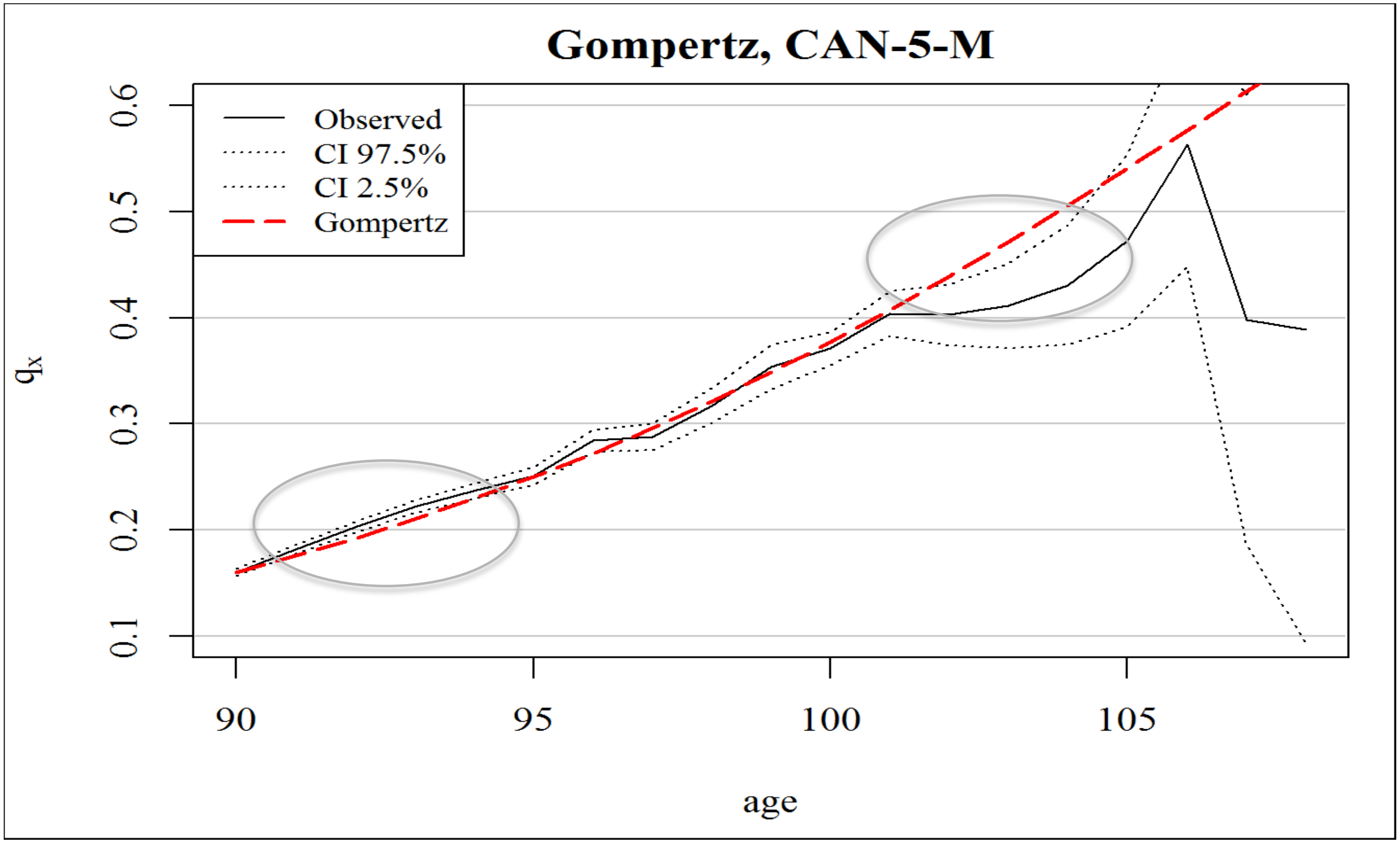
  last age $\hat{\psi} = 130$ from authors, estimated ($\hat{\psi} = 127$ M, 128 F)

  $\ln(\hat{q}_x) = A_1 + A_2 x + A_3 x^2$ $(\Phi \quad 2 \quad 2 \quad \Phi)$

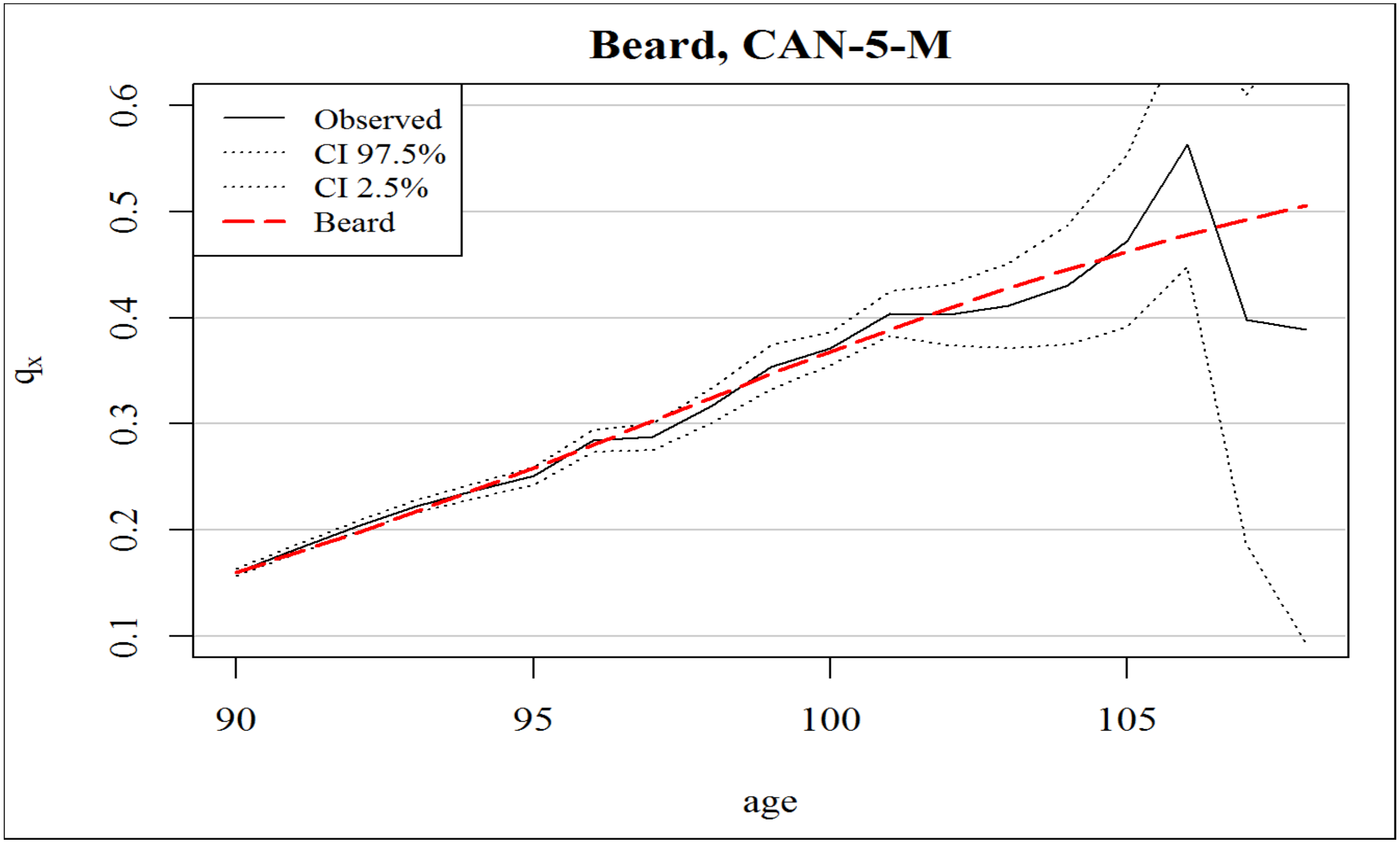
- D&G variant: constraint on $\mu_{\Phi} = 0$ ($q = 0.632$)

  $\ln(\hat{q}_x) = A_1 + A_2 x + A_3 x^2$, last age estimated (120 M, 122 " F)

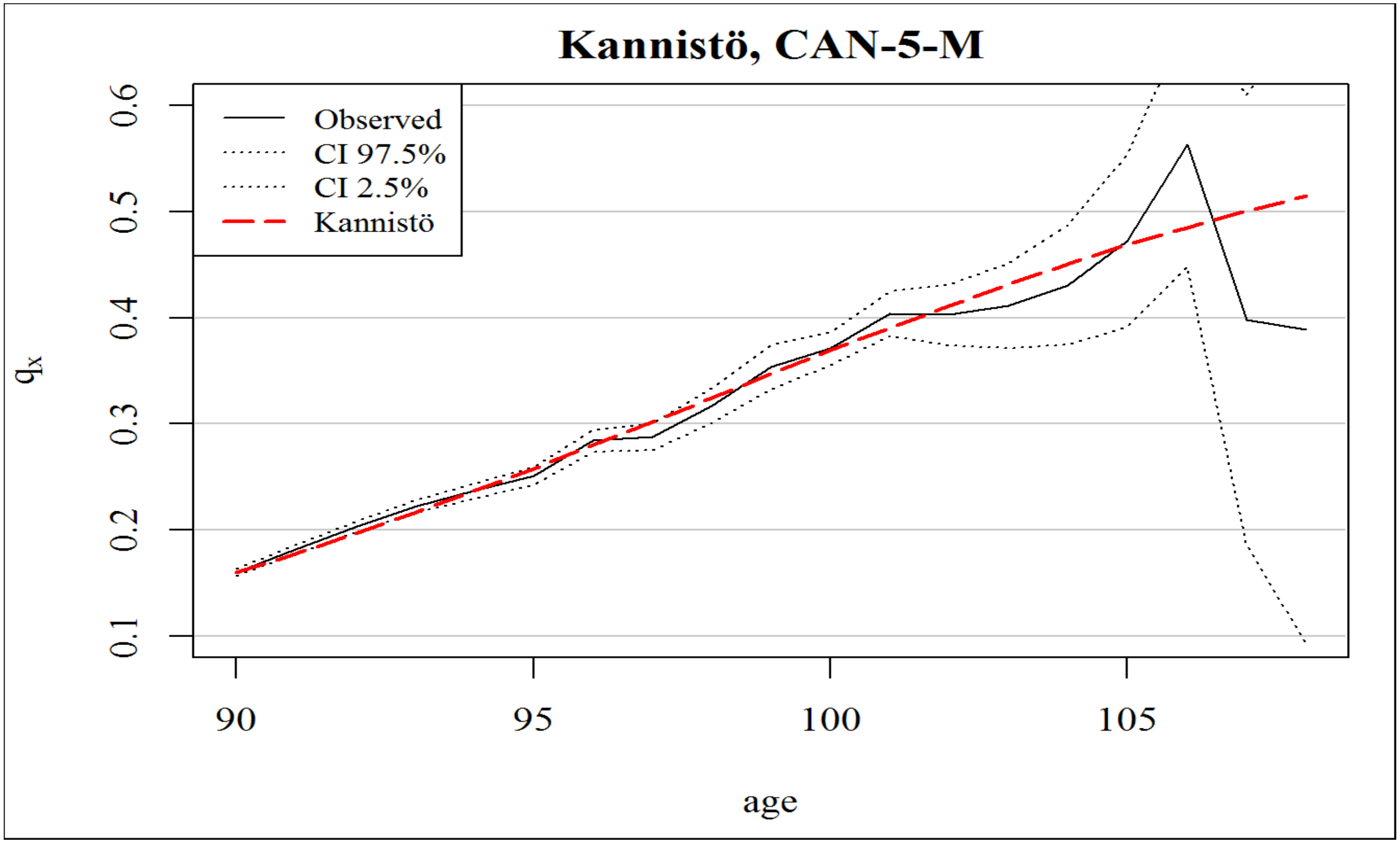
4.3a Illustration, Gompertz CAN-5-M



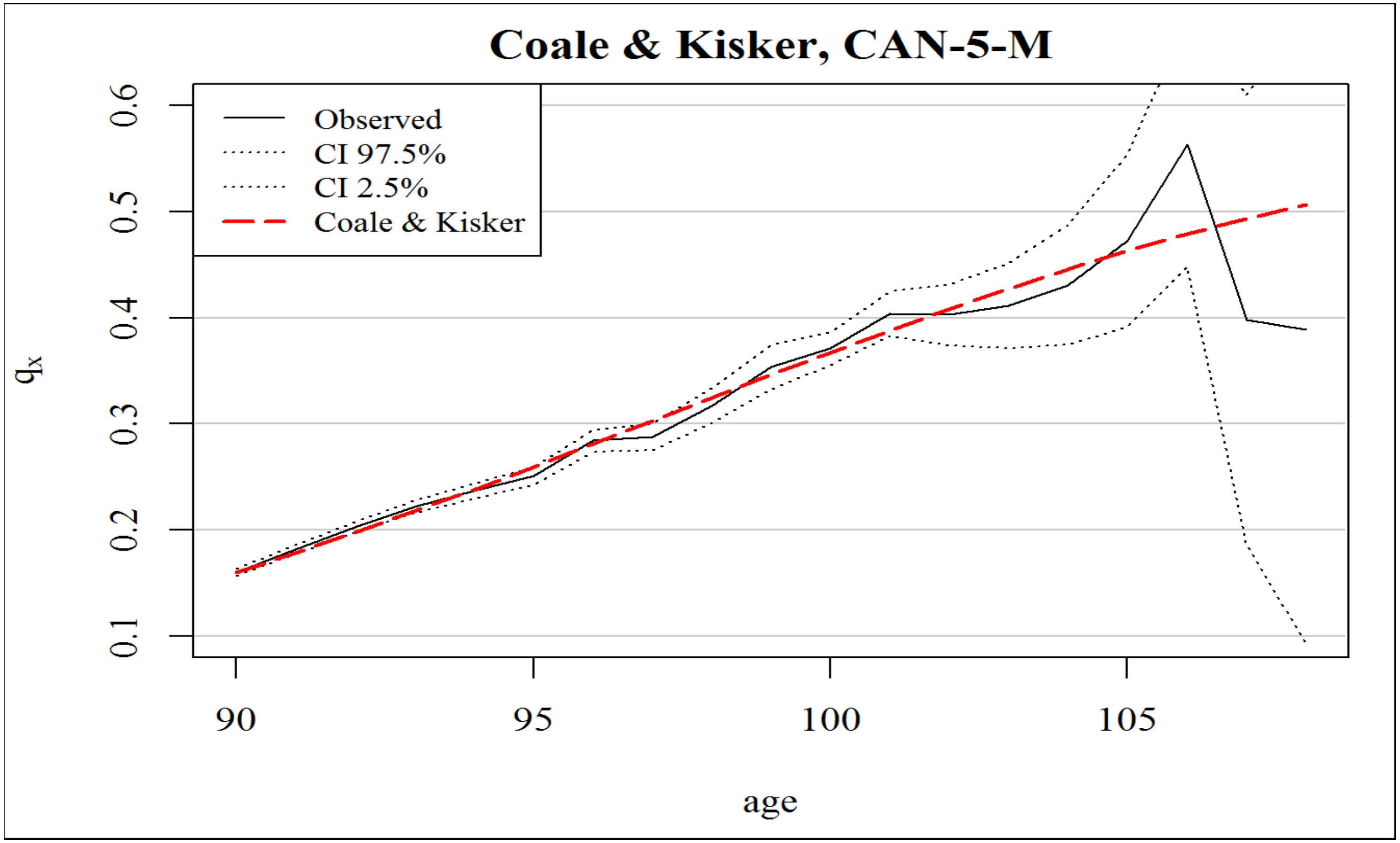
4.3b Illustration, Beard CAN-5-M



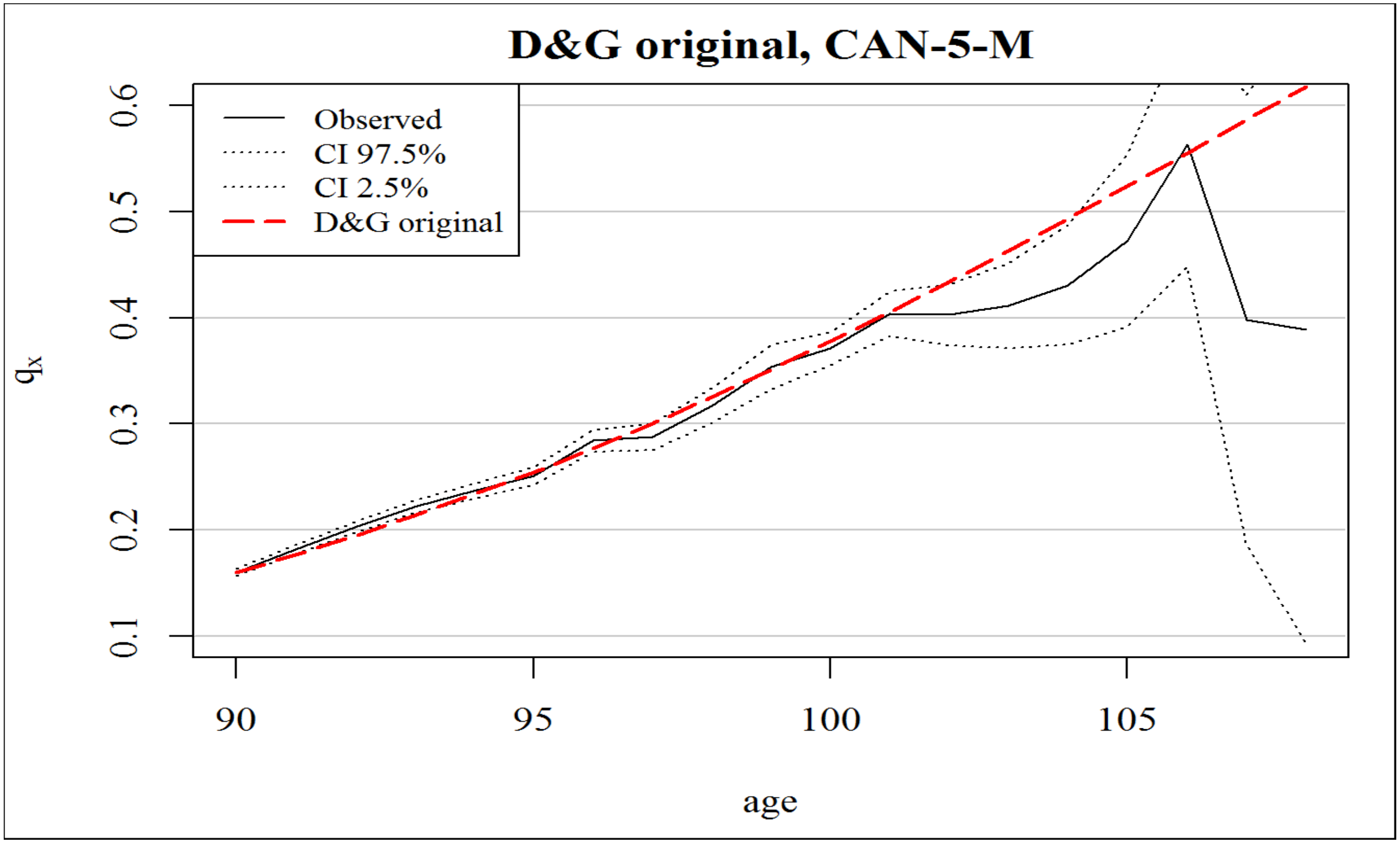
4.3c Illustration, Kannisto CAN-5-M



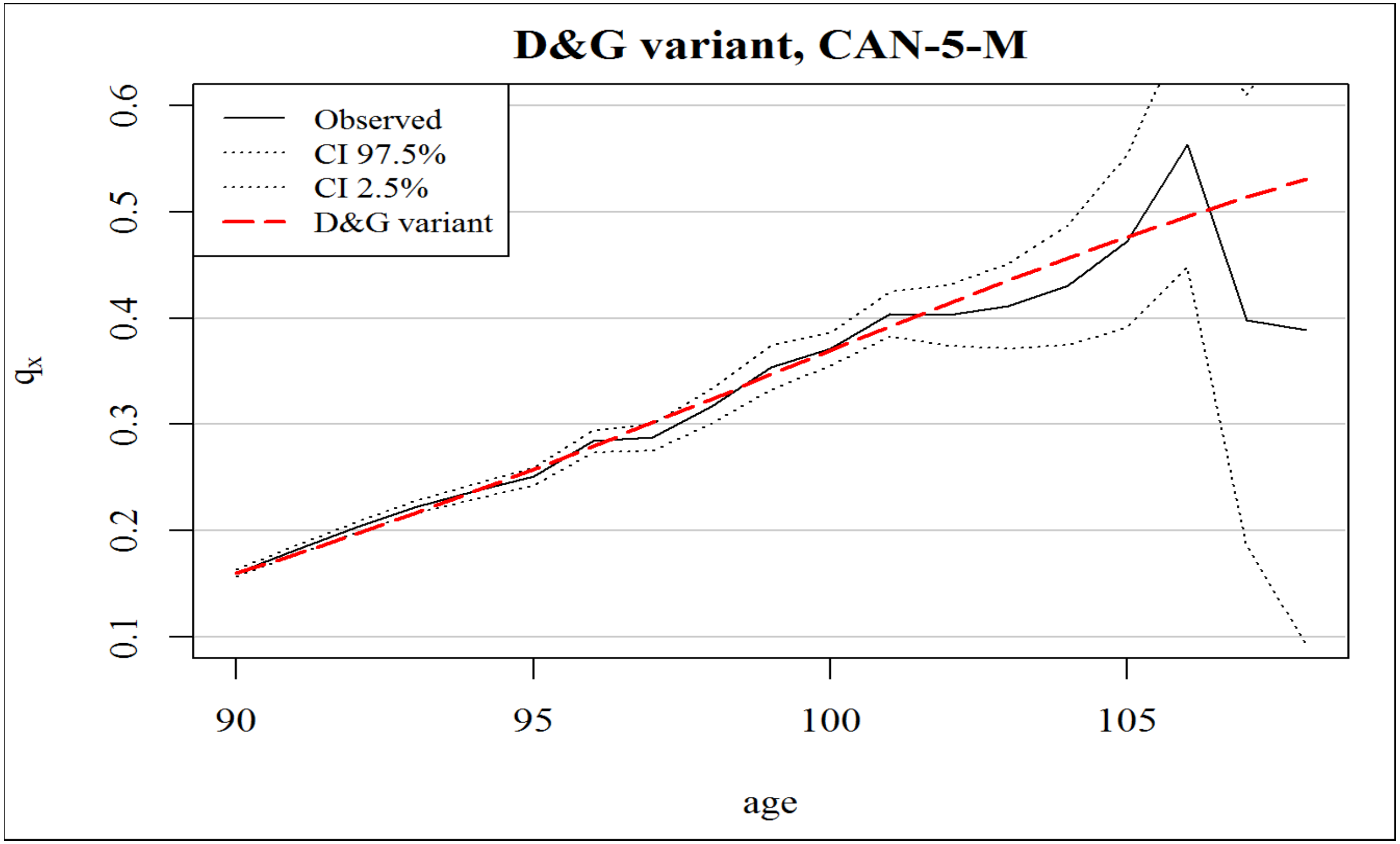
4.3d Illustration, C&K CAN-5-M



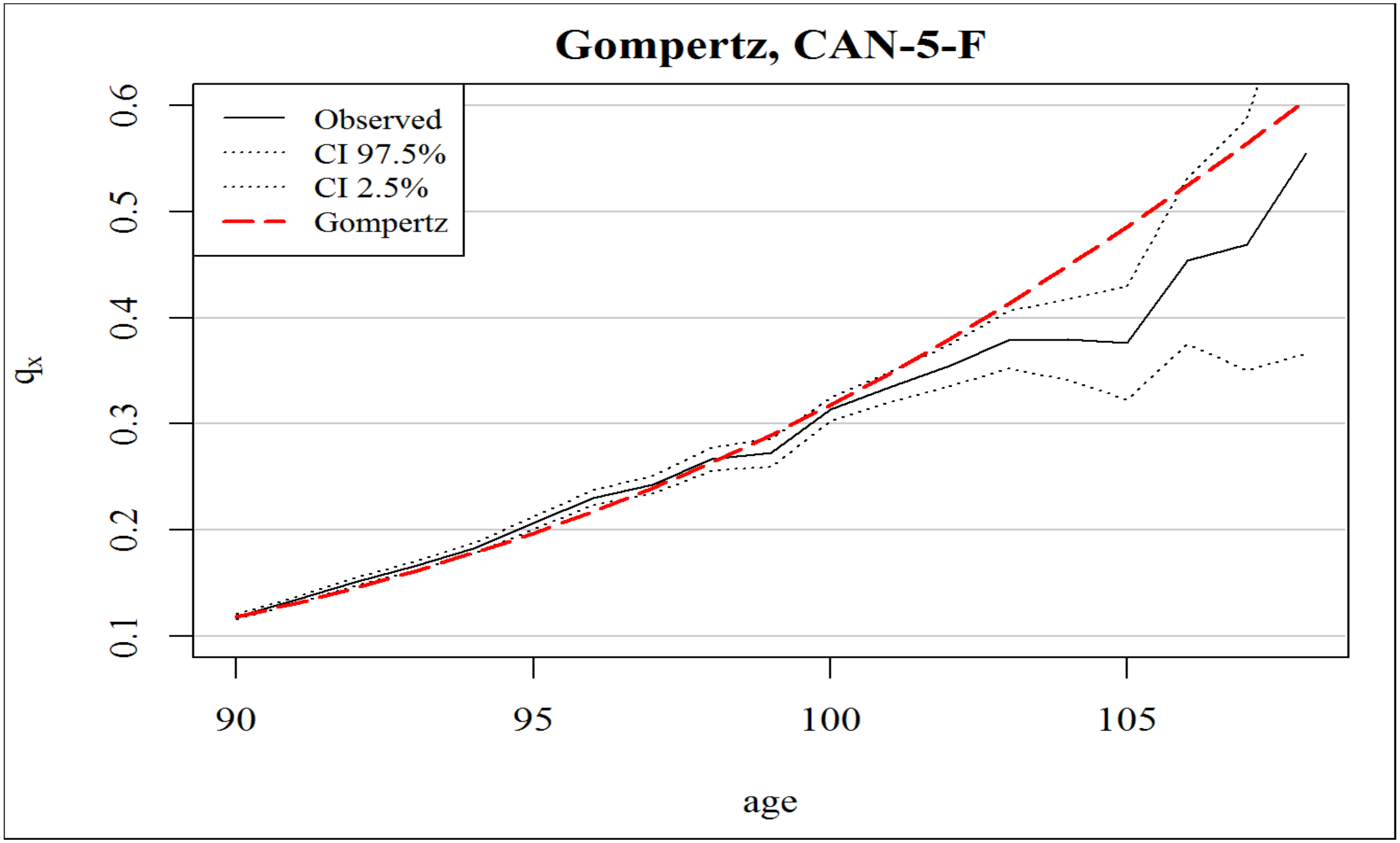
4.3e Illustration, D&G original CAN-5-M



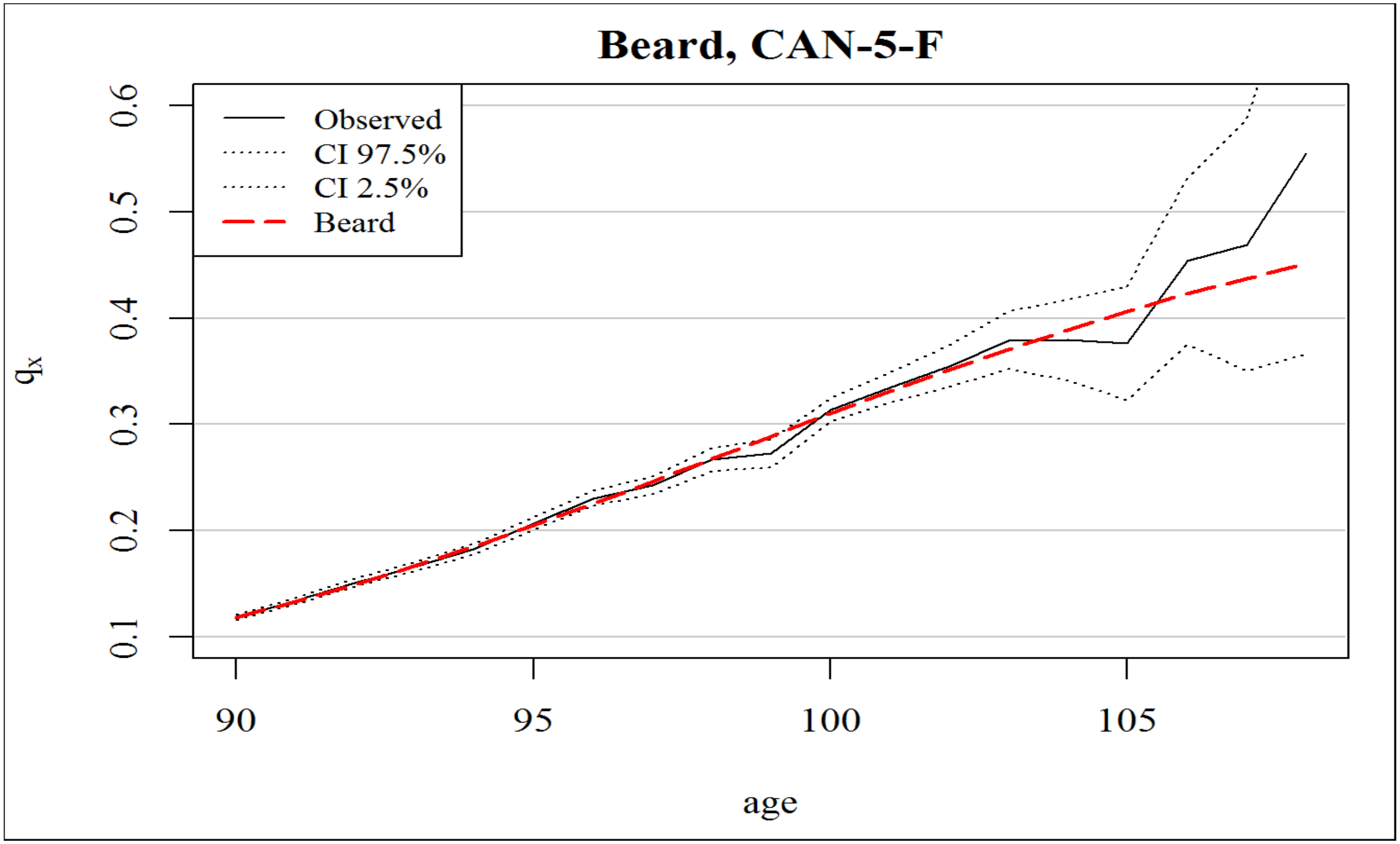
4.3f Illustration, D&G variant CAN-5-M



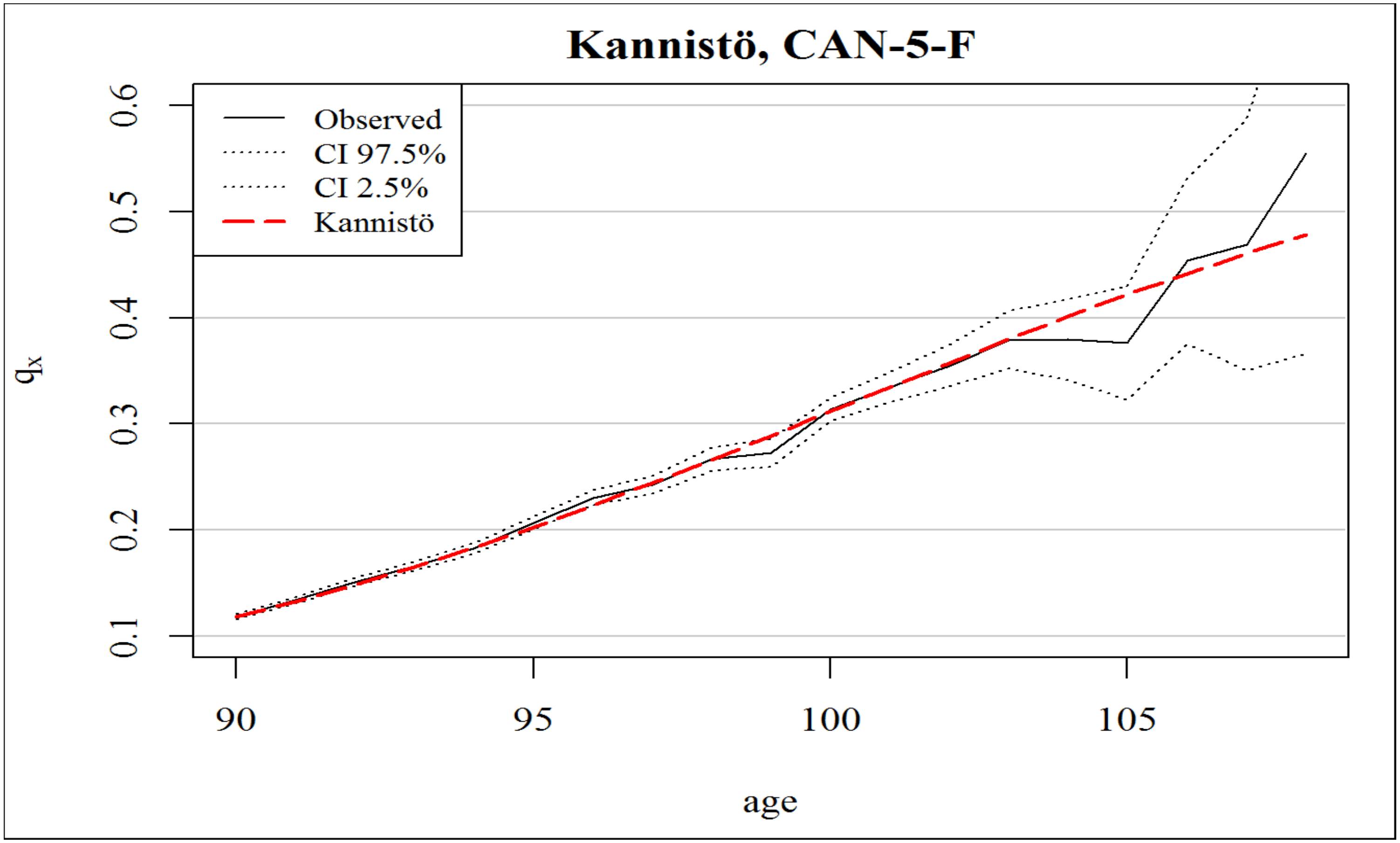
4.4a Illustration, Gompertz CAN-5-F



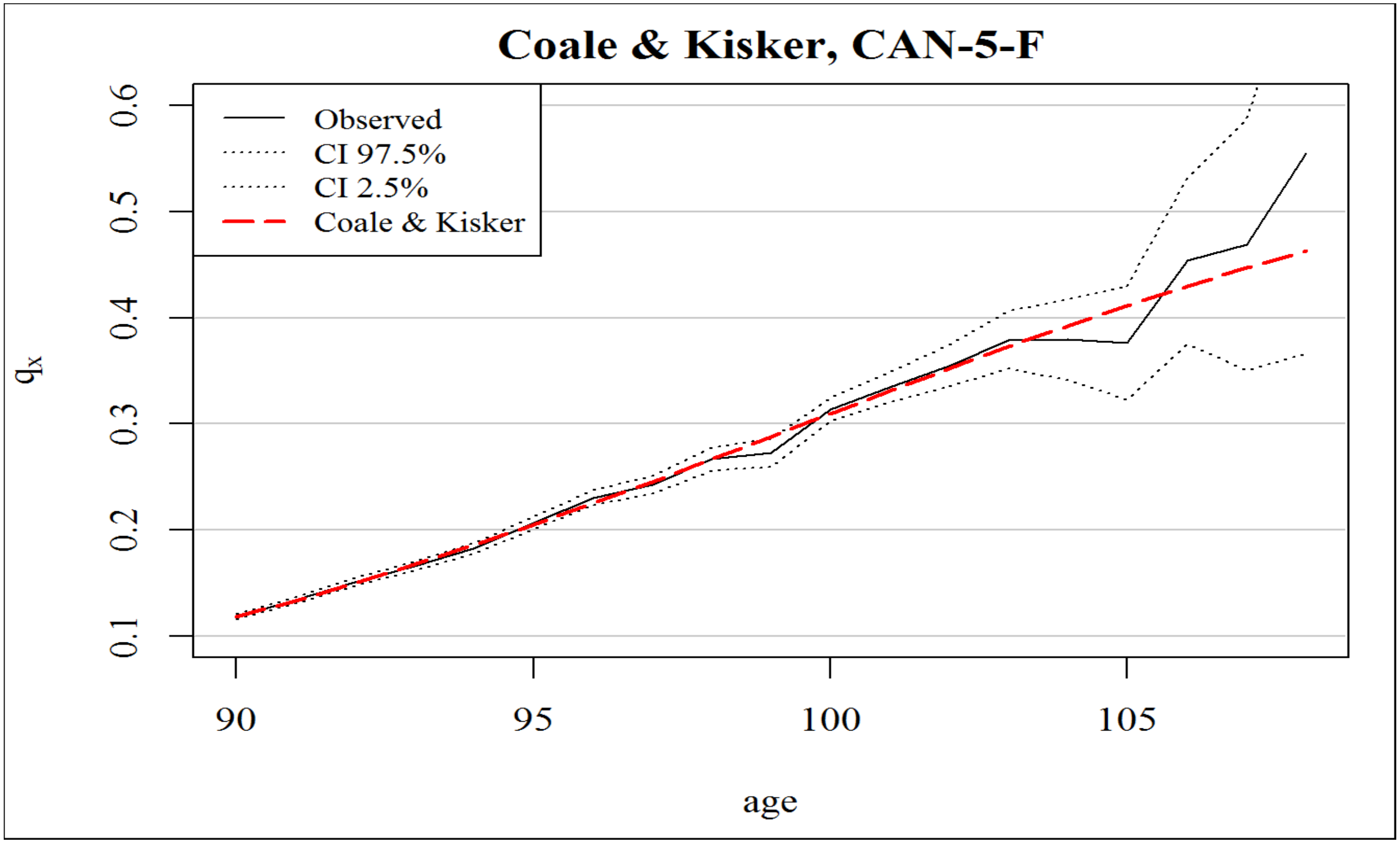
4.4b Illustration, Beard CAN-5-F



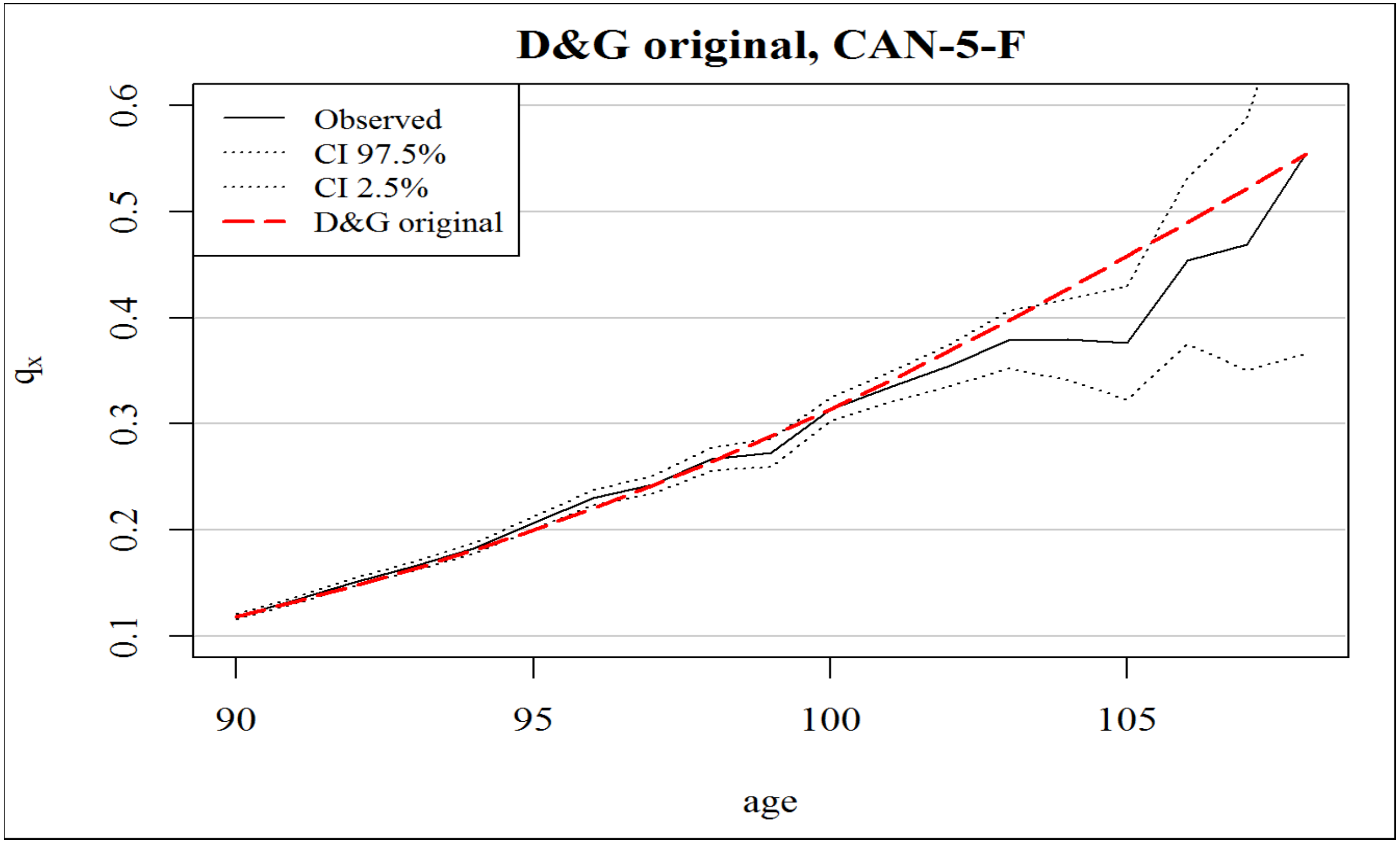
4.4c Illustration, Kannisto CAN-5-F



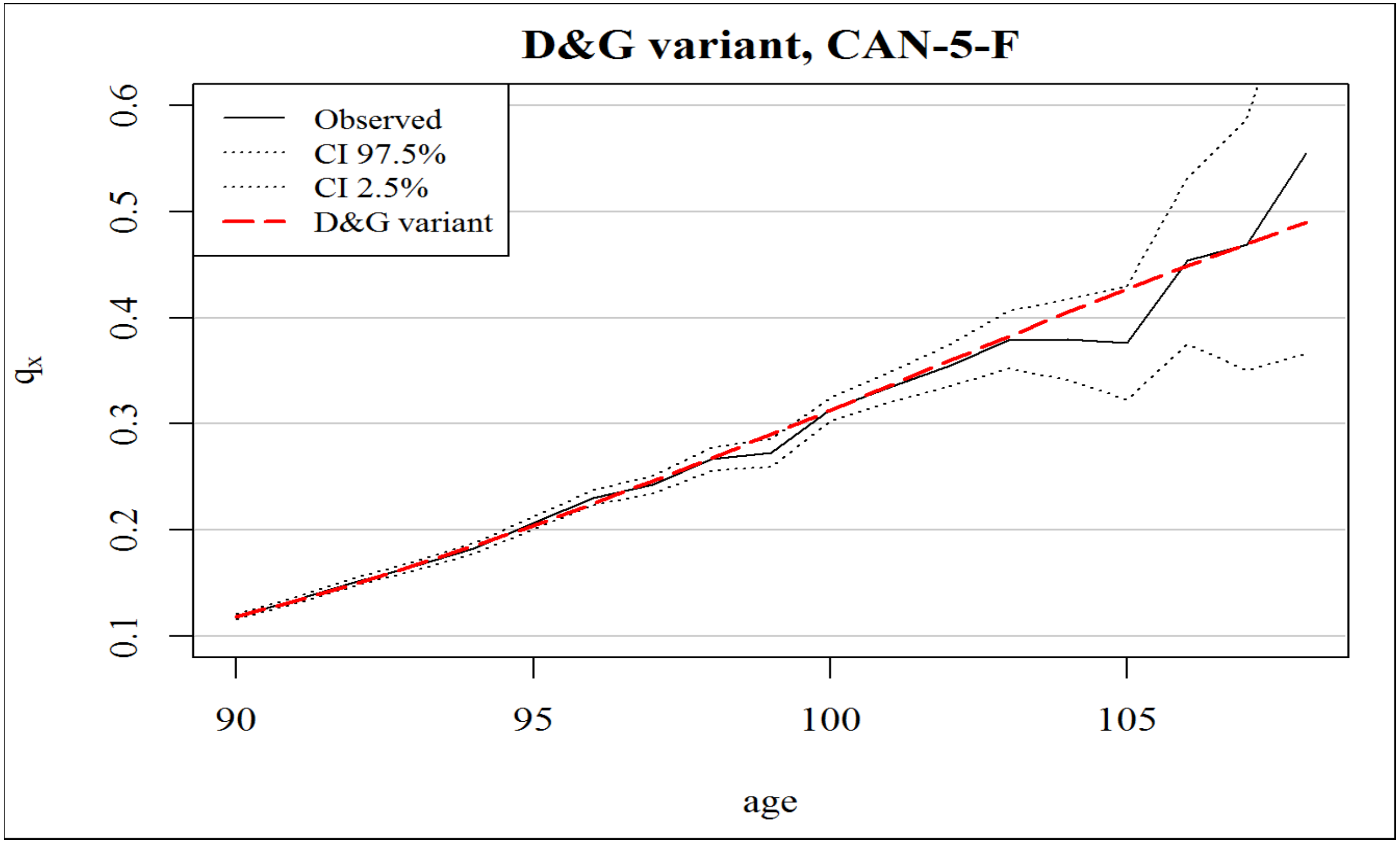
4.4d Illustration, C&K CAN-5-F



4.4e Illustration, D&G original CAN-5-F



4.4f Illustration, D&G variant CAN-5-F



4.5 Ranking of models: fit

- Sum of weighted squared residuals for each model (SSR)
- Weights: inverse of variance of q(x)
- From age 90 to age 105
- Lowest value for C&K
- Closest: Beard
- Gompertz: larger value
- Issue of number of parameters?: no
- Logistic model selected for other tables: OAS study in Canada
- Next slides: SSR figures for males, females

4.6 Ranking of models: Males

Males	Gompertz	Beard	Kannisto	C&K	D&G original	D&G variant
SSR	77.94	26.72	27.53	**24.85**	50.00	26.62
Rank	6	3	4	**1**	5	2

4.7 Ranking of models: Females

Females	Gompertz	Beard	Kannisto	C&K	D&G original	D&G variant
SSR	102.20	14.76	19.76	**14.53**	47.73	18.54
Rank	6	2	4	**1**	5	3

5.1 Materiality: q_x, e_x

- Selected values for probabilities of death
- Comparison to OAS 2012 Mortality Report
- Selected values of complete life expectancy
- Complete life expectancy calculations consistent with constant force of mortality assumption
- Slight difference compared to U.D.D. assumption (<0.5 addition to curtate e_x value)

5.2 Proposed q_x (C&K), CAN-5-M

Age	Proposed qx (C&K)	2007 Version	OAS, 2012
95	0.2594	0.2728	0.2621
100	0.3675	0.3729	0.3671
105	0.4628	0.4594	0.4811
110	0.5275	0.5175	0.5878
115	0.5526	0.5400	0.6669
120	1.0000	1.0000	0.7000

5.3 Proposed q_x (C&K), CAN-5-F

Age	Proposed qx (C&K)	2007 Version	OAS, 2012
95	0.2051	0.2177	0.2109
100	0.3095	0.3215	0.3169
105	0.4116	0.4170	0.4333
110	0.4903	0.4837	0.5410
115	0.5321	0.5100	0.6187
120	1.0000	1.0000	0.6500

5.4 Complete Life Expectancy CAN-5-M

Age	Proposed qx (C&K)	Gompertz on this data set	Beard on this data set	OAS, 2012 (UDD)
95	**2.78**	2.82	2.78	2.80
100	**1.99**	1.86	1.99	2.01
105	**1.54**	1.20	1.54	1.49
110	**1.31**	0.76	1.30	1.17

5.5 Complete Life Expectancy CAN-5-F

Age	Proposed qx (C&K)	Gompertz on this data set	Beard on this data set	OAS, 2012 (UDD)
95	**3.40**	3.24	3.39	3.34
100	**2.37**	2.01	2.37	2.32
105	**1.77**	1.20	1.81	1.69
110	**1.44**	0.69	1.51	1.30

5.6 Comments on materiality

- Difference in models not that large, but C&K fits well this specific data set
- Gompertz: expected probability of death too high, underestimation of life expectancy
- Other models: Very close values for probabilities of death,and curtate life expectancies
- UDD vs. Constant force of mortality: smaller difference at lower ages, but measurable at extreme age (close to 0.02 at age 100…)
- Using a wider time frame over age 100: valid because no evolution of mortality over time and allows estimation with good data quality.

6. Conclusion

- From Gompertz to Coale & Kisker (vs. Logistic)
- Gender Difference: yes
- Income Difference: not over age 90
- Data Source Difference: not over age 90
- Calendar Year Impact: none over age 98-100
- Age 95 to 100: transition from Year Specific to Model
- Age 100 to 115: Model based on 1996-2011 CPM Data, with Coale & Kisker wear-off
- 115 to 119: nearly constant rate, vary by gender
- 120: 1.0

Questions ?

Thank you !

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(very short) Literature Review

- Prof. Robert Bourbeau, U of Mtl, about Canadian data (2006)
  - "There is evidence of a slowing down in the age-associated increase in mortality"
- Robine, Gampe and Vaupel (2005) and Gampe (2010) with International Database on Longevity data
  - Evidence that q_x cap around $q_{110}=0.5$

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