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On the decomposition of life expectancy and limits to life

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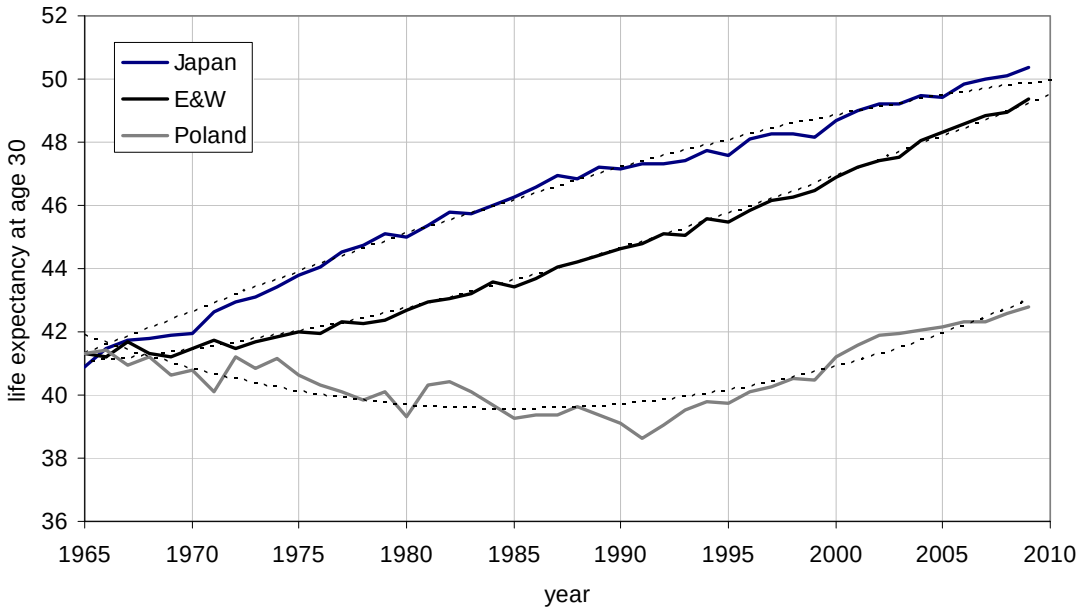
Some key demographic issues

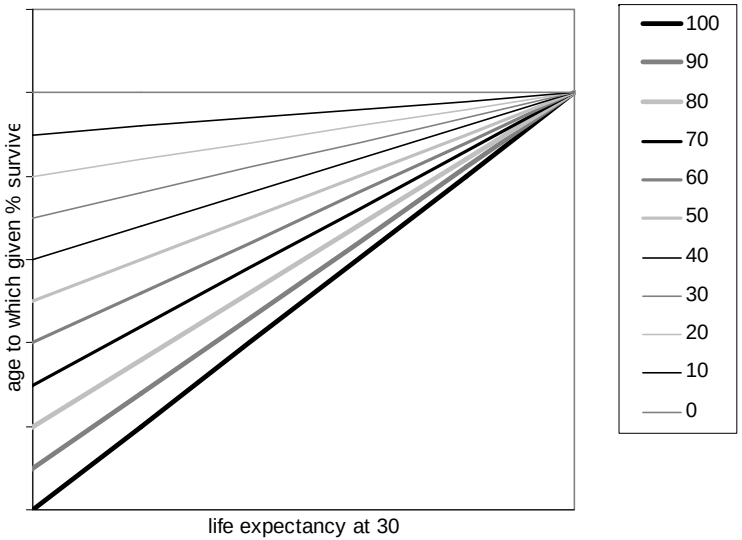
- What do trends in life expectancy in different countries tell us about limits to life expectancy?
- Are there alternatives to the use of mortality as a basis for forecasting life expectancy?
- Why does life expectancy in some countries go down as well as up?
- Why is the US falling behind other major countries?

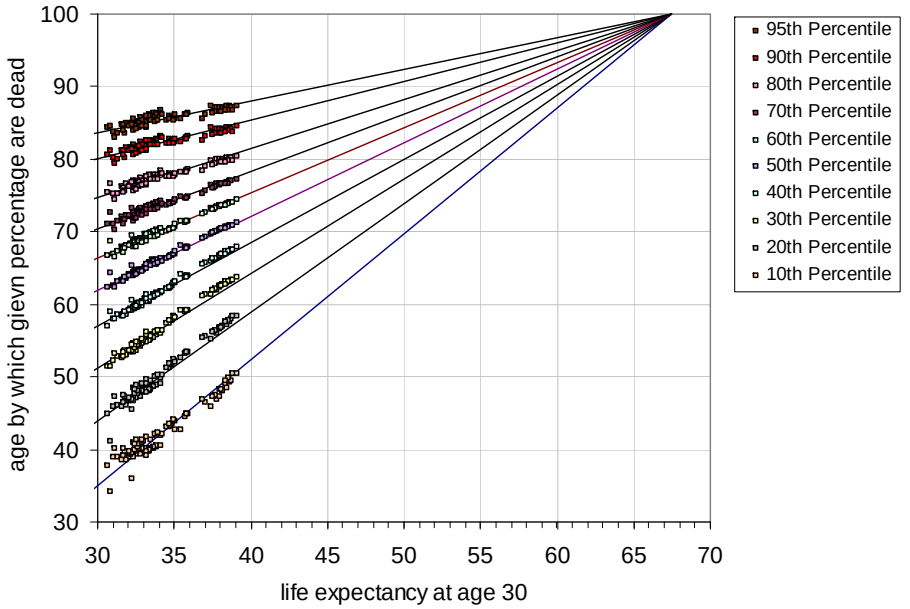
Comparative trends in life expectancy in three countries



Male trends in life expectancy at age 30 in Japan, England and Wales and Poland using quadratic functions (Source – HMD)

Is there evidence for convergence in England and Wales data?





In the convergent case life expectancy proceeds to a maximum. If we take the period 1840 to 1939 there is clear evidence for life expectancy to be on a convergent track although it has a long way to run. In both male and female cases this is 67.5 at age 30.

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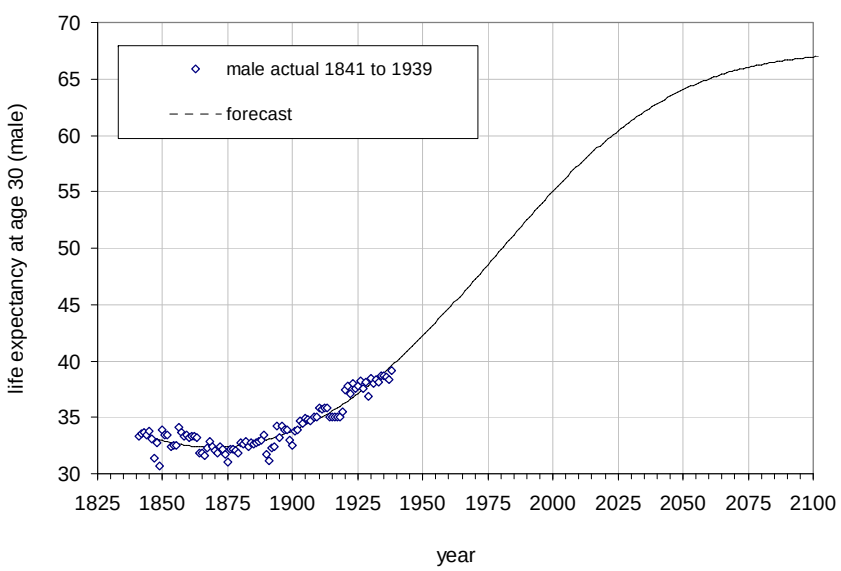
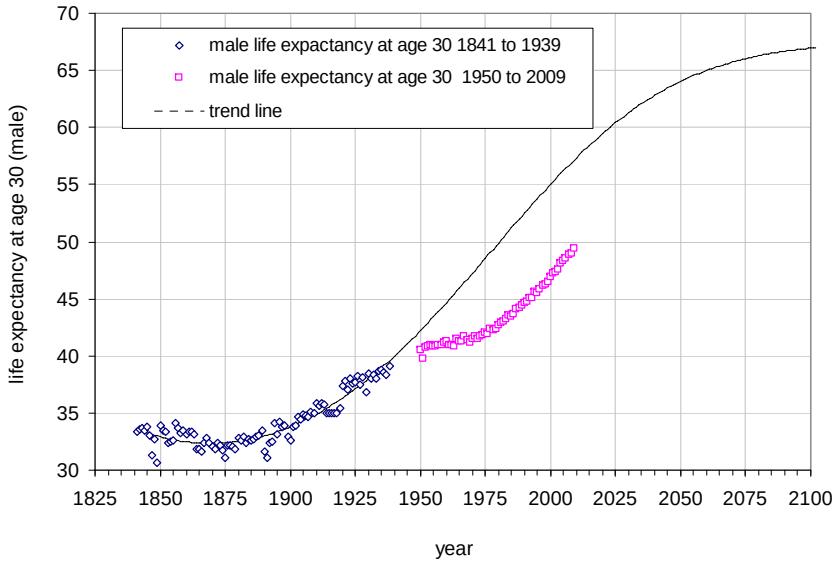
Forecasting life expectancy based on pre-war convergent trend





The trend predicted from 1939 suggests that a maximum would be approach from about 2050 when progress would level out

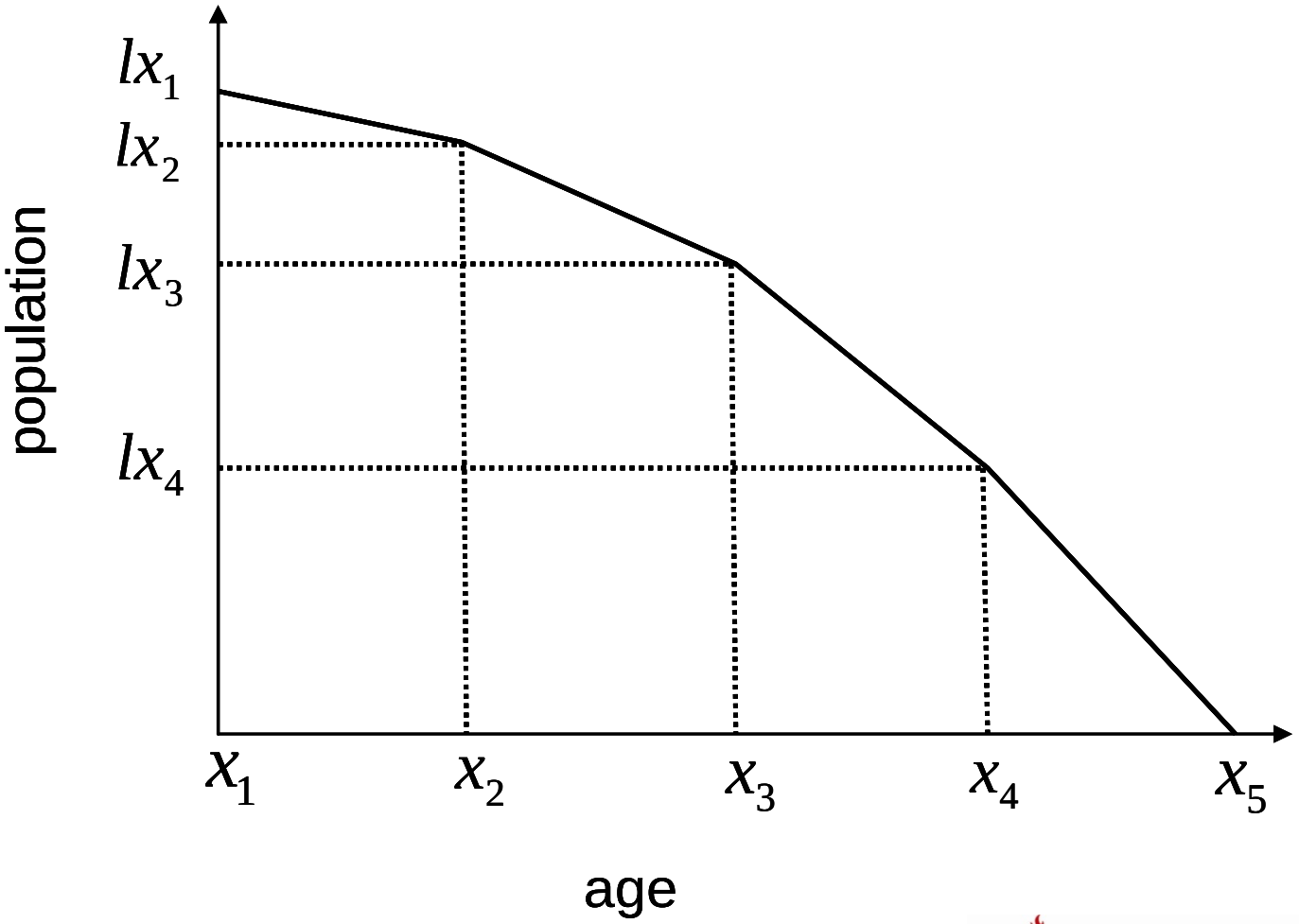
However, the actual trend is severely retarded from about 1950 to 1975 before resuming its steep climb. However, convergent behaviour is replaced by parallel behaviour. The same occurs in females but the retardation effect is less. So hard to estimate upper limit.

Alternative based on partial life expectancy

Motivation

- Create known limits by dividing life course into age intervals
- Identity trends within each based on known upper limits
- Aggregate age intervals to obtain life expectancy between any desired age interval (e.g. 30-100)
- Isolate and focus attention on age intervals where trend are harder to establish (e.g. >100)

Partial life expectancy



Partial life expectancy

- Define $e_{x_n(x_n:x_m)}$ as the future expected life of someone currently aged x_n between the ages of x_n and x_m.

Hence

$$e_{x_1(x_1:x_3)} = \frac{1}{l_{x_1}}\left\{\left(l_{x_2} + \frac{1}{2}\left(l_{x_1} - l_{x_2}\right)\right) + \left(l_{x_3} + \frac{1}{2}\left(l_{x_2} - l_{x_3}\right)\right)\right\}$$

$$= \frac{1}{l_{x_1}}\left\{\sum_{y=x_2}^{x_3} l_y + \frac{1}{2}\left(l_{x_1} - l_{x_3}\right)\right\}$$

And

$$e_{x_1(x_3:x_5)} = \frac{1}{l_{x_1}}\left\{\left(l_{x_4} + \frac{1}{2}\left(l_{x_3} - l_{x_4}\right)\right) + \left(l_{x_5} + \frac{1}{2}\left(l_{x_4} - l_{x_5}\right)\right)\right\}$$

$$= \frac{1}{l_{x_1}}\left\{\sum_{y=x_4}^{x_5} l_y + \frac{1}{2}\left(l_{x_3} - l_{x_5}\right)\right\}$$

So

$$e_{x_1} = e_{x_1(x_1:x_3)} + e_{x_1(x_3:x_5)}$$

Aggregation of partial life expectancy

$$e_{30} = e_{30(30:40)} + \frac{l_{40}}{l_{30}} e_{40(40:50)} + ... + \frac{l_{100}}{l_{30}} e_{100(100:110)}$$

We can add partial life expectancies together to obtain overall life expectancy if we wish. Since we know that life expectancy cannot exceed ten years in a ten year interval we can assess which age groups have reached their limit and which have not. In principle this means we can pin-point where the greatest uncertainty lies and thus focus our attention on the age ranges where future increases in life expectancy will come from

Constructing a population with highest probability of survival

Define: l_x^Z as the number of people alive in population Z at age x

$e^Z_{x_1(x_1:x_2)}$ as the expectation of life between the ages of x_1 and x_2 of someone in population Z currently aged x_1.

Then the expected life of the person aged 30 is:

$$e_{30} = e^A_{30(30:40)} + \frac{l^A_{40}}{l^A_{30}} e^B_{40(40:50)} + \frac{l^A_{40}}{l^A_{30}} \frac{l^B_{50}}{l^B_{40}} e^C_{50(50:60)} \cdots + \frac{l^A_{40}}{l^A_{30}} \frac{l^B_{50}}{l^B_{40}} \frac{l^C_{60}}{l^C_{50}} \frac{l^D_{70}}{l^D_{60}} \frac{l^E_{80}}{l^E_{70}} \frac{l^F_{90}}{l^F_{80}} \frac{l^G_{100}}{l^G_{90}} e^H_{100(100:110)}$$

Male partial life expectancy by leading country 1950 - 2009

	1950	1960	1970	1980	1990	2000	2009
$e_{30(30:40)}$	Sweden	E&W	E&W	E&W	Japan	Sweden	Sweden
$e_{40(40:50)}$	Sweden	Sweden	E&W	E&W	Japan	Sweden	Sweden
$e_{50(50:60)}$	Sweden	Sweden	Sweden	Japan	Japan	Sweden	Sweden
$e_{60(60:70)}$	Sweden	Sweden	Sweden	Japan	Japan	Sweden	Sweden
$e_{70(70:80)}$	Sweden	Sweden	Sweden	Japan	Japan	Japan	Sweden
$e_{80(80:90)}$	Sweden	Sweden	Sweden	Japan	Japan	Japan	Japan
$e_{90(90:100)}$	US	US	Sweden	US	Japan	Japan	Japan
$e_{100(100:110)}$	US	US	US	US	US	Japan	Japan
e_{30} (LC)	Sweden	Sweden	Sweden	Japan	Japan	Japan	Japan

Table showing the leading country by ten year age group in each decade from 1950 onwards in E&W, France, Japan, Sweden and the US

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Male partial life expectancy by lagging country 1950 - 2009

	1950	1960	1970	1980	1990	2000	2009
$e_{30(30:40)}$	Japan	Japan	US	US	US	US	US
$e_{40(40:50)}$	Japan	US	US	France	France	France	US
$e_{50(50:60)}$	Japan	US	US	France	France	France	US
$e_{60(60:70)}$	Japan	US	US	E&W	US	US	US
$e_{70(70:80)}$	Japan	Japan	E&W	E&W	E&W	E&W	US
$e_{80(80:90)}$	Japan	Japan	Japan	E&W	E&W	E&W	Sweden
$e_{90(90:100)}$	Japan	Japan	Japan	Japan	E&W	Sweden	Sweden
$e_{100(100:110)}$	Japan	Japan	Japan	Japan	France	Sweden	Sweden
e_{30} (LC)	Japan	Japan	US	France	US	US	US

Table showing the lagging country by ten year age group in each decade from 1950 onwards in E&W, France, Japan, Sweden and the US

Female partial life expectancy by leading country 1950 - 2009

	1950	1960	1970	1980	1990	2000	2009
$e_{30(30:40)}$	Sweden	Sweden	Sweden	Japan	Japan	Sweden	Sweden
$e_{40(40:50)}$	Sweden	Sweden	Sweden	Japan	Japan	Sweden	Sweden
$e_{50(50:60)}$	Sweden	Sweden	Sweden	Japan	Japan	Japan	Japan
$e_{60(60:70)}$	Sweden	Sweden	Sweden	Japan	Japan	Japan	Japan
$e_{70(70:80)}$	Sweden	Sweden	Sweden	Japan	Japan	Japan	Japan
$e_{80(80:90)}$	US	US	Sweden	France	Japan	Japan	Japan
$e_{90(90:100)}$	US	US	US	US	US	Japan	Japan
$e_{100(100:110)}$	US	US	US	US	US	Japan	Japan
e_{30} (LC)	Sweden	Sweden	Sweden	France	Japan	Japan	Japan

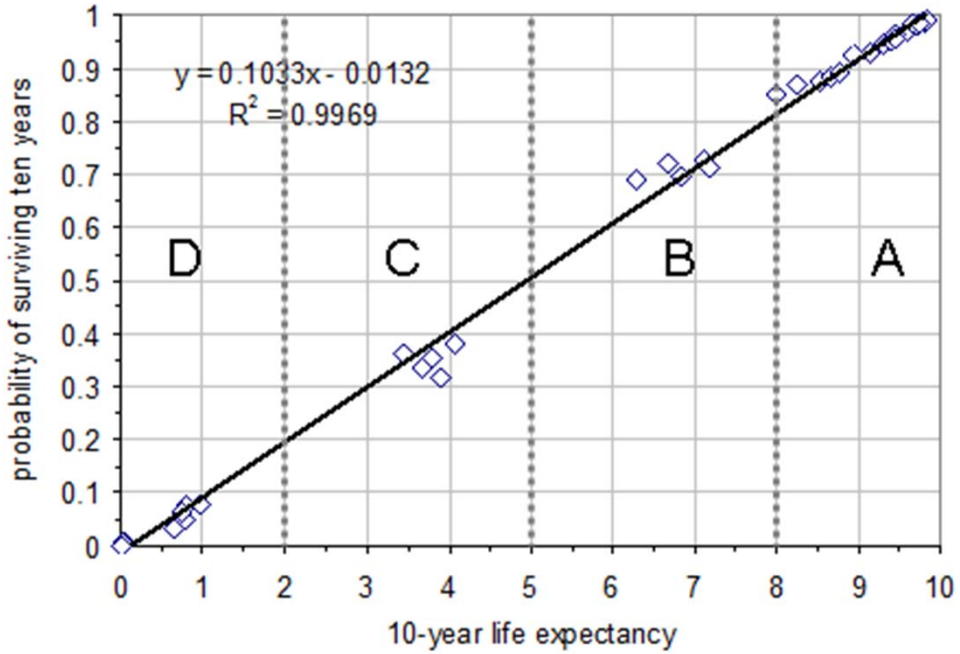
Table showing the leading country by ten year age group in each decade from 1950 onwards in E&W, France, Japan, Sweden and the US

Female partial life expectancy by lagging country 1950 - 2009

	1950	1960	1970	1980	1990	2000	2009
$e_{30(30:40)}$	Japan	Japan	US	US	US	US	US
$e_{40(40:50)}$	Japan	Japan	US	US	US	US	US
$e_{50(50:60)}$	Japan	Japan	US	US	US	US	US
$e_{60(60:70)}$	Japan	Japan	US	US	US	US	US
$e_{70(70:80)}$	Japan	Japan	Japan	E&W	E&W	US	US
$e_{80(80:90)}$	Japan	Japan	Japan	E&W	E&W	US	US
$e_{90(90:100)}$	Japan	Japan	Japan	Japan	E&W	E&W	Sweden
$e_{100(100:110)}$	Japan	Japan	Japan	Japan	Sweden	Sweden	Sweden
e_{30}	Japan	Japan	Japan	E&W	E&W	US	US

Table showing the lagging country by ten year age group in each decade from 1950 onwards in E&W, France, Japan, Sweden and the US

Empirical relationship between probability of survival and 10-year life expectancy



Probability of survival in the next ten years versus life expectancy in the next ten years: Key A from age 30 to 60 a greater than 85% of surviving a decade; B from age 70 to 80 ~ 70% chance; C from age 80 to 90 ~ 35% chance; D from age 90 to 100 ~ 5% chance

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Partial life expectancies for Japanese females

From age 30	1950	1960	1970	1980	1990	2000	2009	*2030*	*2050*
$e_{30(30:40)}$	9.8	9.9	9.9	10.0	10.0	10.0	10.0	*10.0*	*10.0*
$e_{30(40:50)}$	9.2	9.6	9.8	9.9	9.9	9.9	9.9	*10.0*	*10.0*
$e_{30(50:60)}$	8.4	9.1	9.4	9.6	9.7	9.7	9.7	*9.9*	*9.9*
$e_{30(60:70)}$	6.9	7.9	8.4	9.0	9.2	9.3	9.4	*9.7*	*9.9*
$e_{30(70:80)}$	4.4	5.4	6.2	7.3	8.0	8.4	8.7	*9.4*	*9.7*
$e_{30(80:90)}$	1.4	1.9	2.5	3.7	4.9	5.9	6.5	*8.4*	*9.2*
$e_{30(90:100)}$	0.1	0.2	0.3	0.5	1.0	1.9	2.5	*5.7*	*8.1*
$e_{30(100:100+)}$	0.0	0.0	0.0	0.0	0.0	0.1	0.2	*1.3*	*4.9*
e_{30}	40.2	44.1	46.5	49.9	52.7	55.2	57.0	*64.3*	*71.8*

Table shows 10-year partial life expectancies for Japanese females from1950 with forecasts for 2030 and 2050. Bottom row shows full life expectancy at age 30

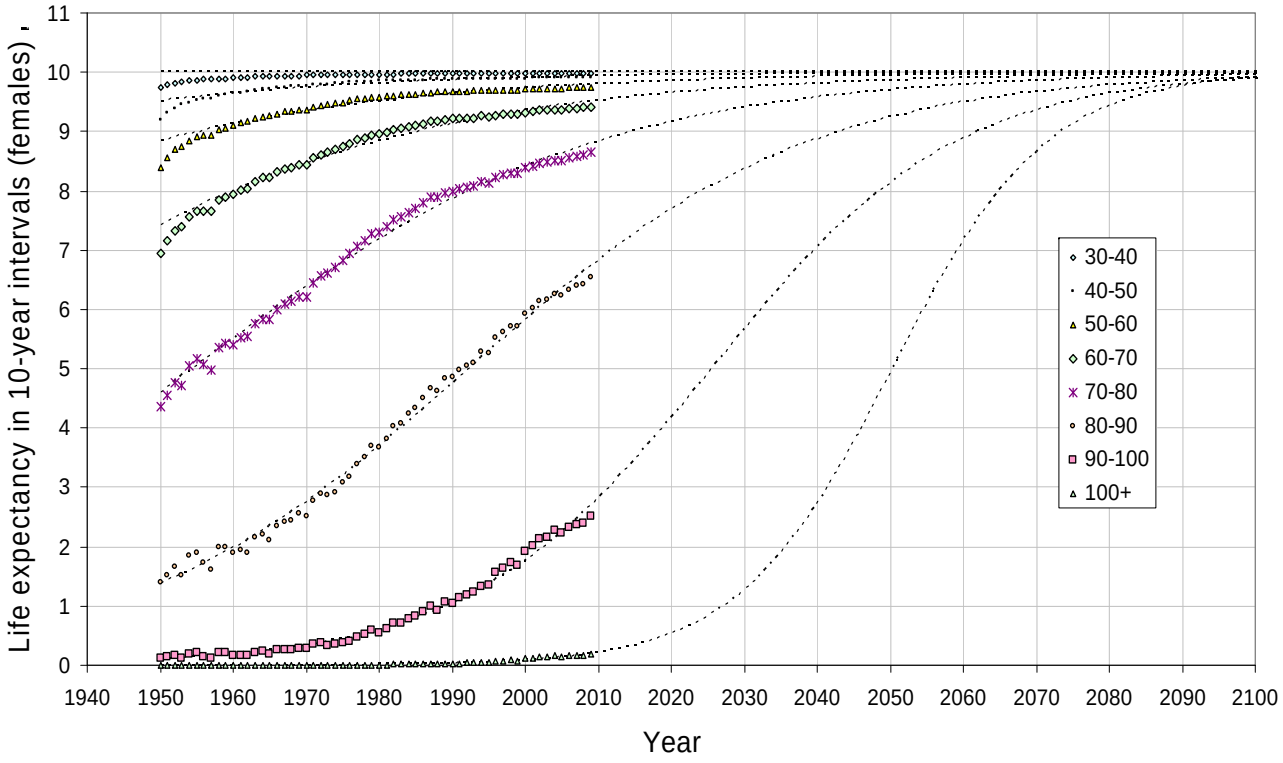
Fitting trend lines to partial life expectancies

The trend lines for Japanese females are based on the following equations:

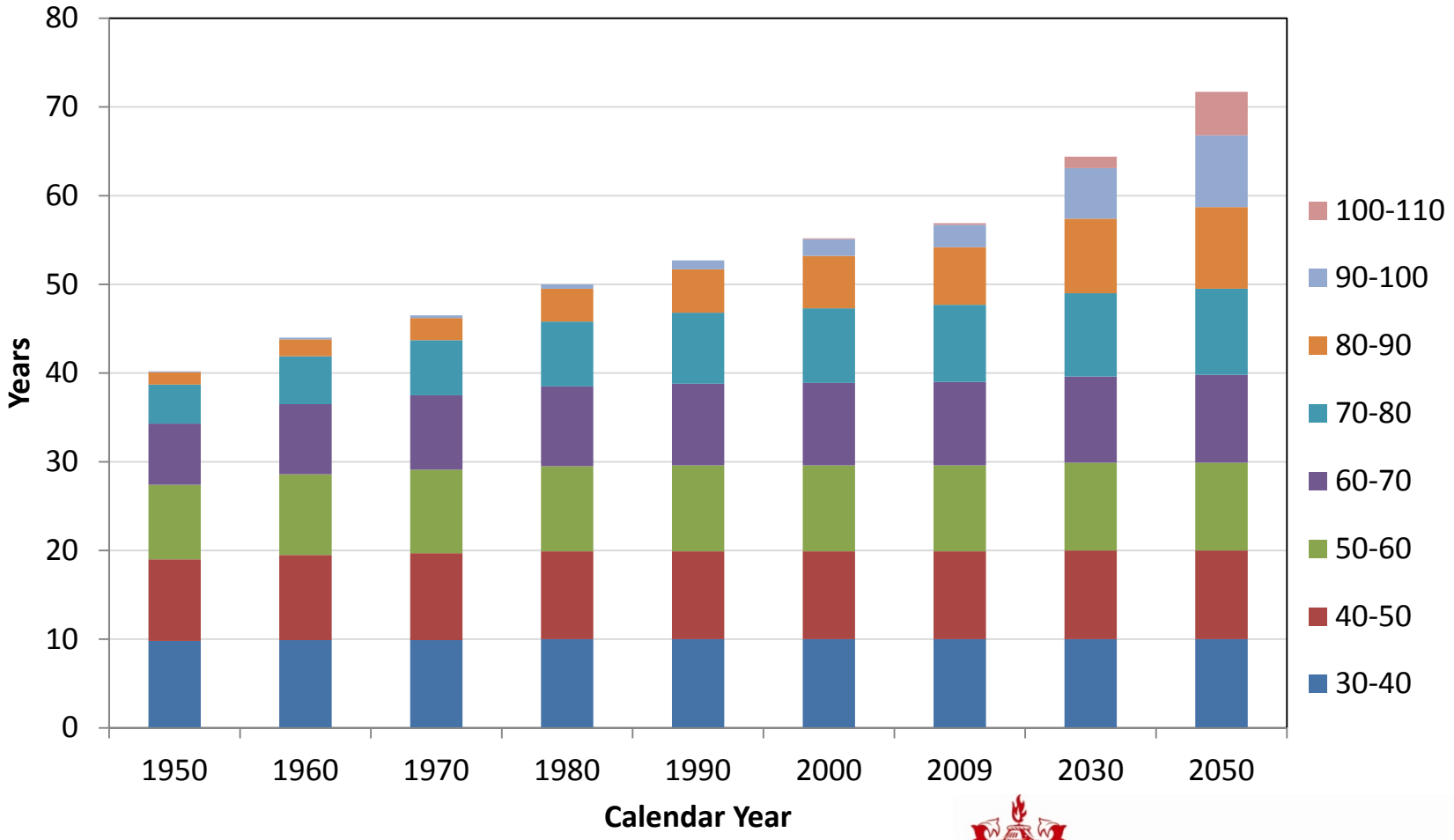
$$y_i = \frac{Ae^{f_i(t)}}{1+e^{f_i(t)}}$$

$$f_i(t) = a_i + b_i t$$

The equation fits most of the series considered very accurately although this is not the only variant



Contribution from each decade of life to Japanese Life Expectancy at 30



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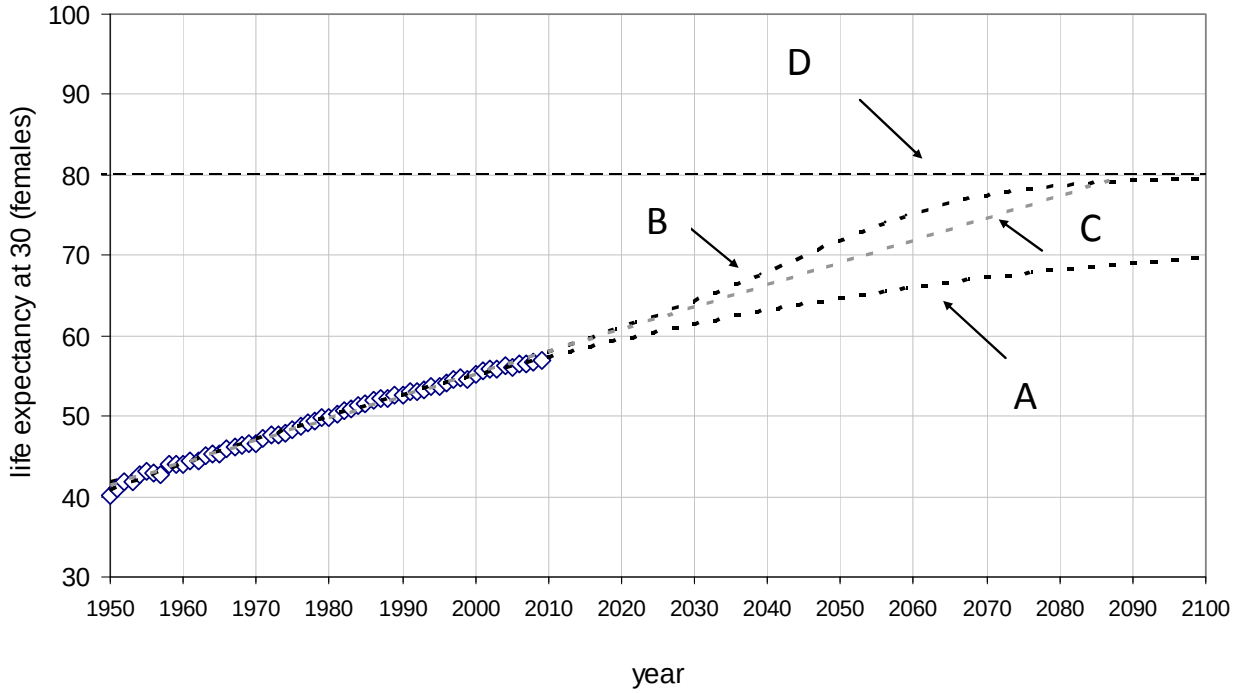
How trend lines compare



A comparison of trends in Japanese female life expectancy at age 30 based on three forecasting approaches: the quadratic, straight line and limiting models.

Key: A quadratic trend; B based on limiting model with upper limit of D; C based on a straight line trend.

Conclusions

- Life expectancy is a crucial measure of human development but if when making forecasts trend lines are unbounded they can become unrealistic
- Partial life expectancies offer a way around this, at least in part
- Retrospective analysis of trends in partial life expectancy show important strategic shifts with the US falling to the bottom and Japan rising to the top

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