Portfolio Choice with Life Annuities under Probability Distortion

Wenyuan Zheng James G. Bridgeman

Department of Mathematics
University of Connecticut

The 49th Actuarial Research Conference, 2014

Outline

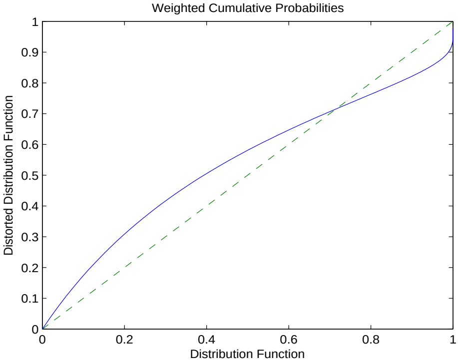
Outline

The Need for Behavioral Economics

- Policyholder's behaviors affect insurance companies' operations
- Behavioral Economics help understand policyholder's behaviors

Probability Distortion

- People overweight small probabilities and underweight large probabilities



Outline

Literature Review

- Yarri (1965)
  - Optimal for an individual without a bequest motive to fully annuitize
- Milevsky and Young (2007)
  - Realistically incorporated mortality-contingent payout annuities (e.g. DB plan)
- Wang and Young (2012)
  - Commutable life annuities to maximize the lifetime utility
- Young and Zariphopoulou (1999)
  - Derive stochastic differential equation for a distorted probability via stochastic differential games

Outline

Goal

- Behavioral Economics
  - Probability distortion
- Portfolio Choice
  - Investment, consumption and annuitization strategy
- Continuous-time setting
- Maximize the lifetime utility
- Commutable life annuity

Market

- A riskless asset
- A risky asset
- Commutable life annuities
  - A single premium immediate annuity with a surrender option

New Probability Distortion

- Typical distortion function
  - $w(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}$
- Why difficult to apply?
  - Hard to derive its stochastic differential equation
- We propose a new distortion function
  - $w(p) = 1 - \frac{1}{1-\delta \cdot \ln(1-p)}, \delta > 1$

Weibull Distribution

- Why Weibull distribution for stock price?
  - Explicit hazard function
- Original SDE
  - $dX_s = [-X_s{}^{\gamma} + \gamma X_s{}^{\gamma-\beta} \frac{\sigma^\beta}{\beta}]ds + (2X_s{}^{\gamma-\beta+1} \frac{\sigma^\beta}{\beta})^{\frac{1}{2}} dB_s$
- Distorted SDE
  - $dX_s = [-X_s{}^{\gamma} + \gamma X_s{}^{\gamma-\beta} \frac{\sigma^\beta}{\beta} + 2X_s{}^{\gamma}(-1 + \frac{2\delta}{1+\delta \frac{X_s}{\sigma}{}^{\beta}})]ds + (2X_s{}^{\gamma-\beta+1} \frac{\sigma^\beta}{\beta})^{\frac{1}{2}} dB_s$
- β: shape parameter
  σ and γ: scale parameters

Outline

Model

- Wealth dynamics
  $dW_s = [r(W_s - \Pi_s) - {\Pi_s}^{\gamma} + \gamma {\Pi_s}^{\gamma-\beta} \frac{\sigma^\beta}{\beta} + 2{\Pi_s}^{\gamma}(-1 + \frac{2\delta}{1+\delta \frac{\Pi_s}{\sigma}^{\beta}}) - C_s + A_s]ds + (2{\Pi_s}^{\gamma-\beta+1} \frac{\sigma^\beta}{\beta})^{\frac{1}{2}} dB_s$
- Value function
  $U(W, A) = \sup_{\pi_s, c_s} \mathbb{E}[\int_0^\infty e^{-(r+\lambda)s} u(c_s) ds | W_0 = W, A_0 = A]$

Model

- HJB equation
$(r+\lambda)U = (rW_s + A_s)U_w + \max_{\pi_s}\{[-r\Pi_s - {\Pi_s}^{\gamma} + \gamma {\Pi_s}^{\gamma-\beta}\frac{\sigma^{\beta}}{\beta} + 2{\Pi_s}^{\gamma}(-1 + \frac{2\delta}{1+\delta\frac{\Pi_s}{\sigma}^{\beta}})]U_w + {\Pi_s}^{\gamma-\beta+1}\frac{\sigma^{\beta}}{\beta}U_{ww}\} + \max_{c_s}(\frac{c_s^{1-\gamma}}{1-\gamma} - cU_w)$

Numerical Method

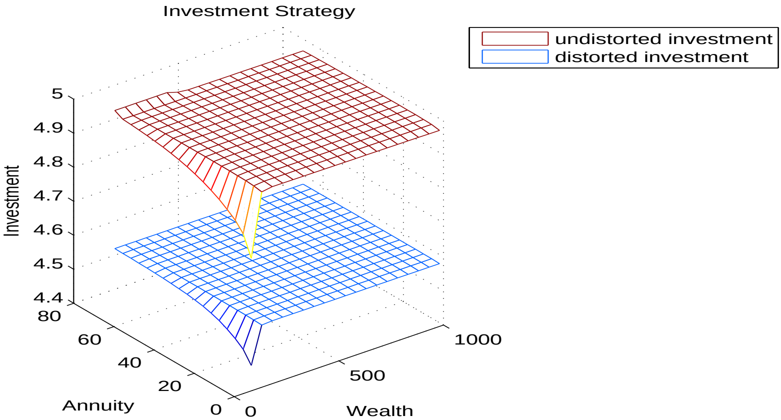
- $U_w(i,j+1) = U_w(i,j) + [W(2) - W(1)] \cdot U_{ww}(i,j+1)$
- $U(i,j+1) = U(i,j) + [W(2) - W(1)] \cdot U_w(i,j+1)$

Parameters

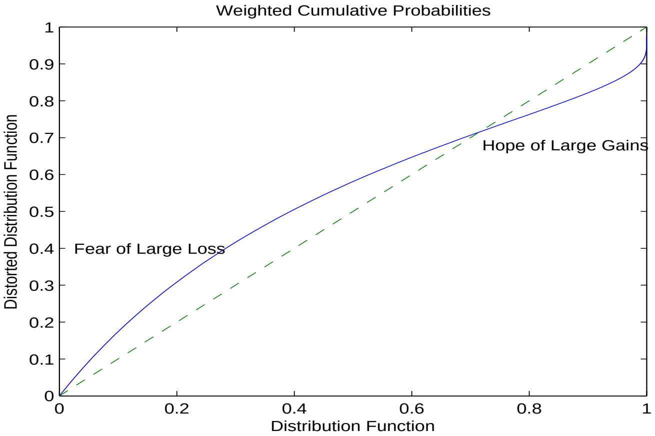
- $r = 0.04$
- $\lambda = 0.04$
- $\beta = 1$
- $\sigma = 5$
- $\gamma = 2$
- $\alpha = 2.5$
- $p = 0.2$
- $\delta = 2$

Outline

Investment

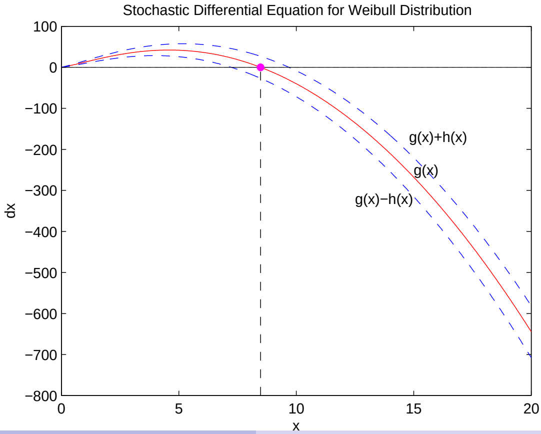

Investment Strategy
undistorted investment
distorted investment
Investment
5
4.9
4.8
4.7
4.6
4.5
4.4
Annuity
80
60
40
20
0
Wealth
0
500
1000

Investment


Weighted Cumulative Probabilities
Distorted Distribution Function
Distribution Function
Fear of Large Loss
Hope of Large Gains
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

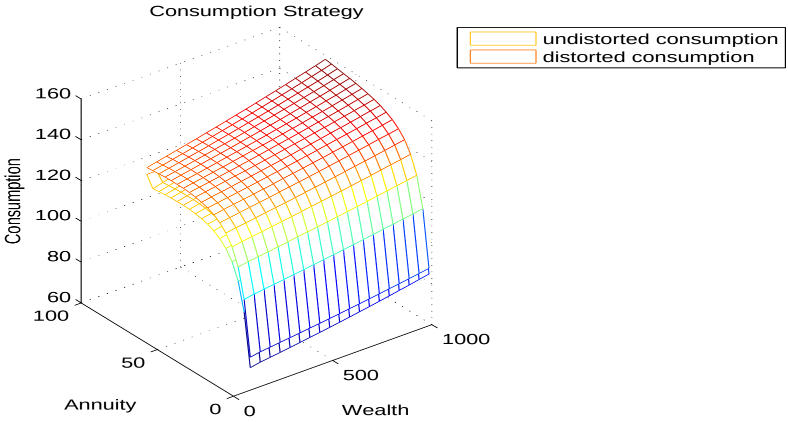
Investment

- $-{X_s}^{\gamma} + \gamma {X_s}^{\gamma-\beta}\frac{\sigma^{\beta}}{\beta} + 2{X_s}^{\gamma}(-1 + \frac{2\delta}{1+\delta \frac{X_s}{\sigma}^{\beta}}) > 0$

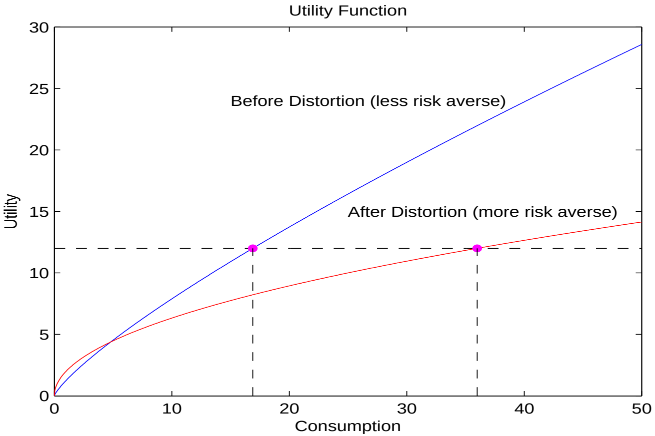


Outline

Consumption



Consumption



Outline

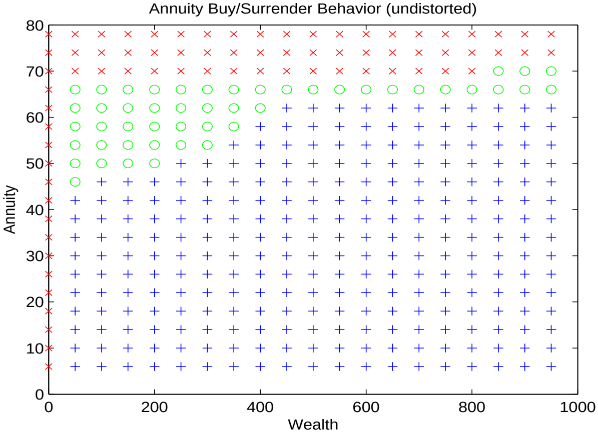
Annuitization Strategy

Behavior	Utility		
Buy	U(A+4,W-50)		
Do nothing		U(A,W)	
Surrender			U(A-4,W+40)

Annuitization Strategy

Undistorted Case

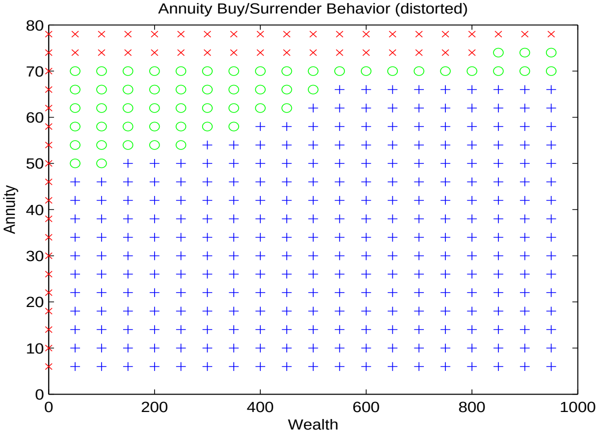
+: buy
○: do nothing
×: surrender



Annuitization Strategy

Distorted Case

+: buy
○: do nothing
×: surrender



Annuitization Strategy

- Need more annuities to against fear
  - Stop buying annuity at a higher level
  - Begin surrendering annuity at a higher level
- Different z_0: critical ratio of wealth-to-annuity
  - An unique z_0 in Wang and Young (2012)
- Behavior pattern

Annuity $\longrightarrow$	Surrender
	Surrender $\rightarrow$ Do nothing
	Do nothing
	Do nothing $\rightarrow$ Buy
	Buy
	Wealth $\longrightarrow$

Illustration

W=500 A=62

	No Distortion	Distortion
Stock	4.97	4.57
Bond	495.00	445.43
Consumption	138.42	145.36
Annuitization	Do nothing	Buy

One year later...

No Distortion	W=468 A=62
Distortion	W=422 A=66

Illustration

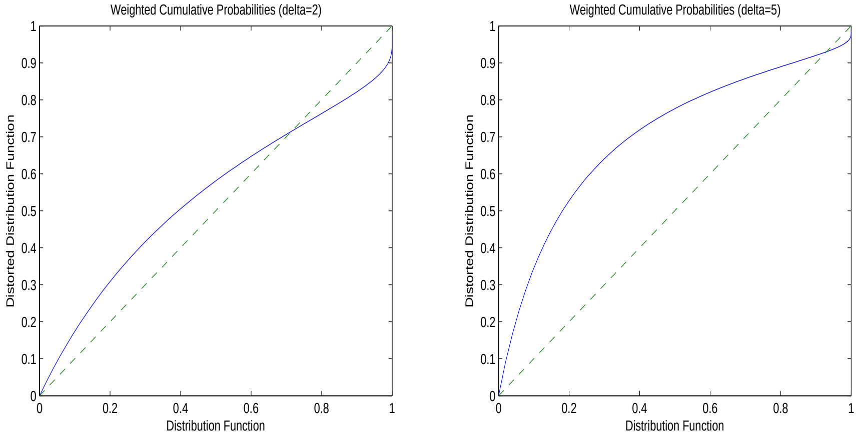
W=1000 A=74

	No Distortion	Distortion
Stock	4.96	4.57
Bond	995.04	995.43
Consumption	149.04	157.72
Annuitization	Surrender	Do nothing

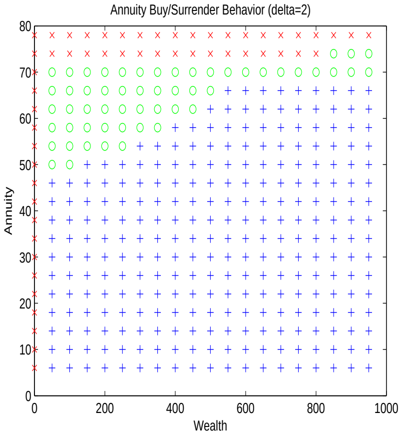
One year later...

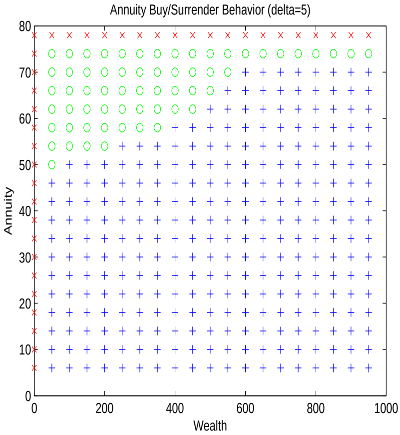
No Distortion	W=1030 A=70
Distortion	W=998 A=74

Sensitivity Analysis



Sensitivity Analysis





Conclusion

- Probability distortion brings more fear
- To against fear
  - Invest less on risky asset
  - Consume more
  - Need more annuity (also support more consumption)
- Contribution of this work
  - A new distortion function
  - Weibull distribution for stock price
  - Annuitization behavior available for each pair of (Wealth, Annuity)

Thank you!