On the Interaction between Transfer Restrictions and Crediting Strategies in Guaranteed Funds

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Financial support from the Society of Actuaries under the CAE research grant is greatly appreciated.

Crediting Strategies

- Contract Descriptions
  - Employees deposit money at regular intervals into a designated account
  - The employee can direct the funds to a number of different accounts
  - Subject to only a few restrictions, they can rebalance their portfolio whenever they want.

Questions…

- Why do insurance companies credit anything other than short term rates on what is (essentially) a demand account?
  - Transfer Restrictions
  - Market Value Adjustments
  - Difficulty switching companies
- What should they do?
- What *do* they do?
- How do policyholders respond?

The Model

- The “game” proceeds as follows. At time t:
  - IC picks r_c, the rate he will credit for the next time period.
  - PH picks his allocation, ω_{t+1}, which becomes a state variable for the next period.
  - PP buys assets, which become state variables for the next period.

The Model

- BDT Interest Rate Model
  - Calibrated with 0.14 volatility
- Outcomes:
  - Zero Sum under Q (PV of Book Value Profit)
  - IC likes Q, PH likes utility under P

Propositions

- 2.1 - IC's asset purchase strategy is independent of his crediting strategy and independent of PH's choices.
- 2.2 - IC is indifferent to his asset strategy.
- 2.3 - If there are no transfer restrictions, IC will credit a rate $r_c < r_{t,1}$ and PH will allocate $\omega_{t+1} = 1$ or IC will credit $r_c = r_{t,1}$ and PH will allocate $0 \leq \omega_{t+1} \leq 1$.
- 2.4 - At any given time and state with $\omega_t = 1$, the expected present value of future book profits under Q is the market value of the assets less the book value of the assets. Specifically, the expectation at initiation of the contract is 0.

Proposition 2.5

- In the presence of transfer restrictions, the only reasonable allocations in the period t+1 are $\omega_{t+1} = 0$ and $\omega_{t+1} = (1 - x)\omega_t + x$ (or complete indifference to allocation). The decision of which allocation to choose is independent of the current allocation.

Proof of Prop 2.5

- Imagine the PH has three independent accounts:
  - A guaranteed account of $(1 - x)(1 - \omega_t)$ which must remain in the guaranteed account and cannot be affected by the PH's current choice.
  - A guaranteed account of $x(1 - \omega_t)$ currently allocated to the guaranteed account but fully allocatable in the next period.
  - A money market account of ω_t currently allocated to the money market account but fully allocatable in the next period.

The Optimal Strategies:

- 2.6 - In the first period, the policyholder is free to invest at any value of $0 \leq \omega_1 \leq 1$. If there are transfer restrictions, IC will credit a rate $r_c \leq r_{crit}$ where $r_{crit} \geq r_{1,1}$ and depends on time and state. PH will allocate $\omega_1 = 1$ if $r_c < r_{crit}$ and $0 \leq \omega_1 \leq 1$ if $r_c = r_{crit}$.

The Optimal Strategies:

- 2.7 -The value of r_{crit} is independent of the state variable ω_t.
- 2.8 - If $\omega_t > 0$, IC should set $r_c = 0$.
- 2.9 – If IC credits an interest rate larger than r_{crit}, and PH can borrow and lend at prevailing rates outside the pension plan, an arbitrage opportunity exists for PH.

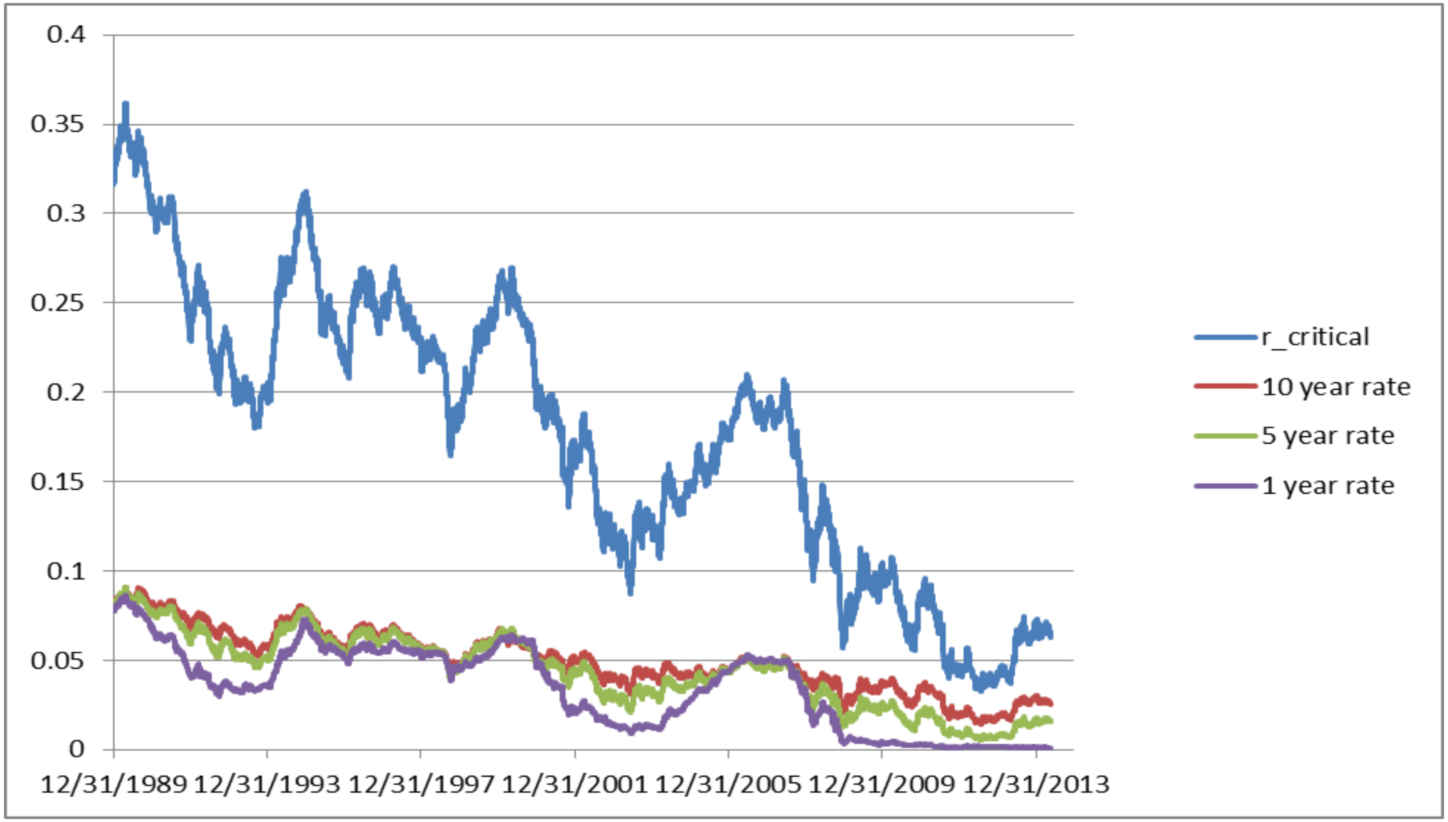
Utility Maximizing PolicyHolders

- Most results still hold even when PH attempts to maximize expected utility under the P measure.
- Risk-Averse Policyholders under P tend to prefer the “trap” strategy to the “money market” strategy since it works better in falling rate scenarios and worse in rising rate scenarios.
- IC credits $r_{crit}^{P} \leq r_c \leq r_{crit}$

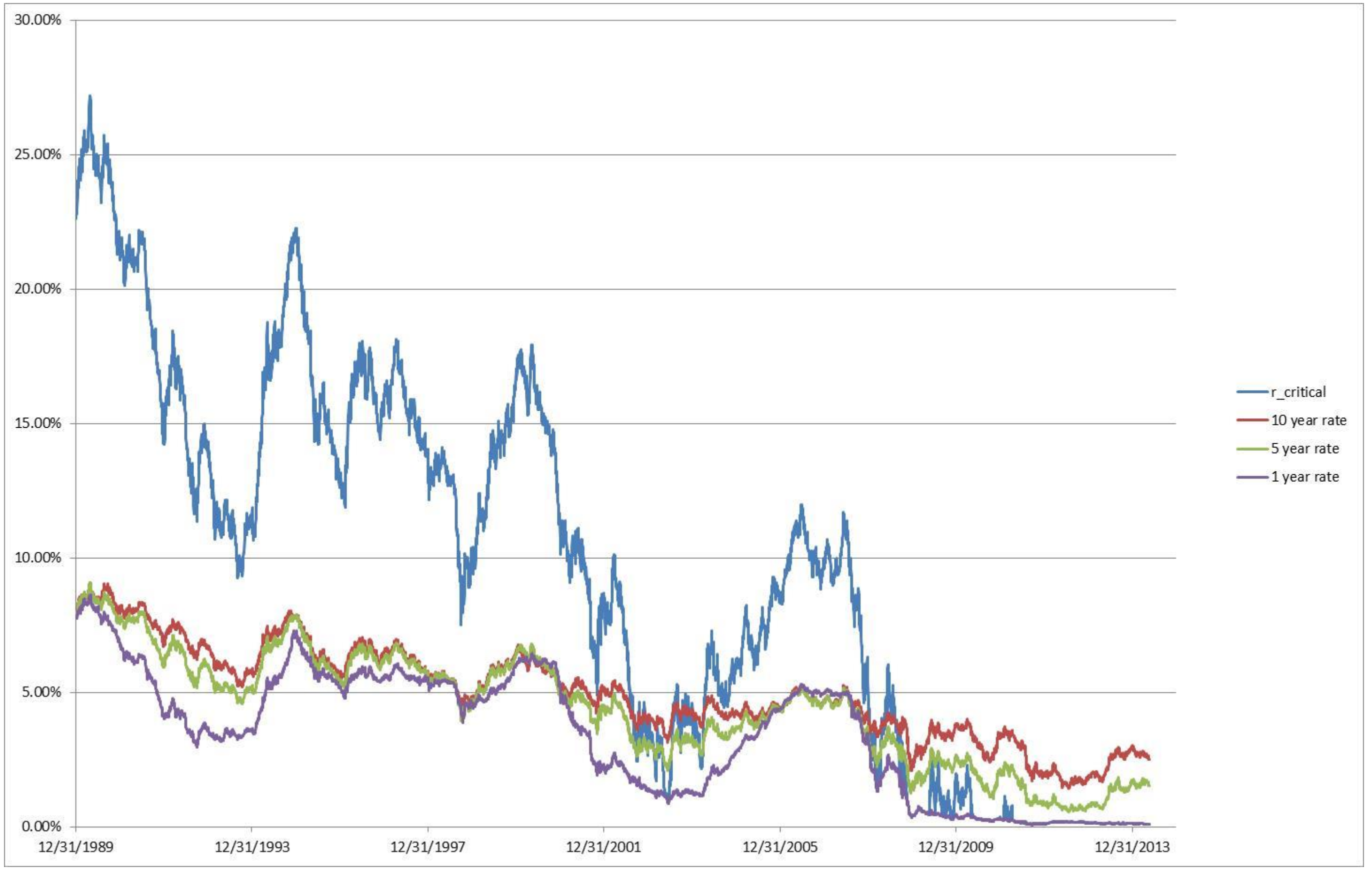
Effect of Minimum Guarantees

- 2.9 Restated - If $\omega_t > 0$, PP should set $r_c = r_{\min}$.
- It is possible for $r_{\min}$ to exceed r_{crit} in which case PH transfers to guaranteed fund (Option Value)
- Value at initiation is not “0”.
- Utility under P may still allow IC to make a profit.

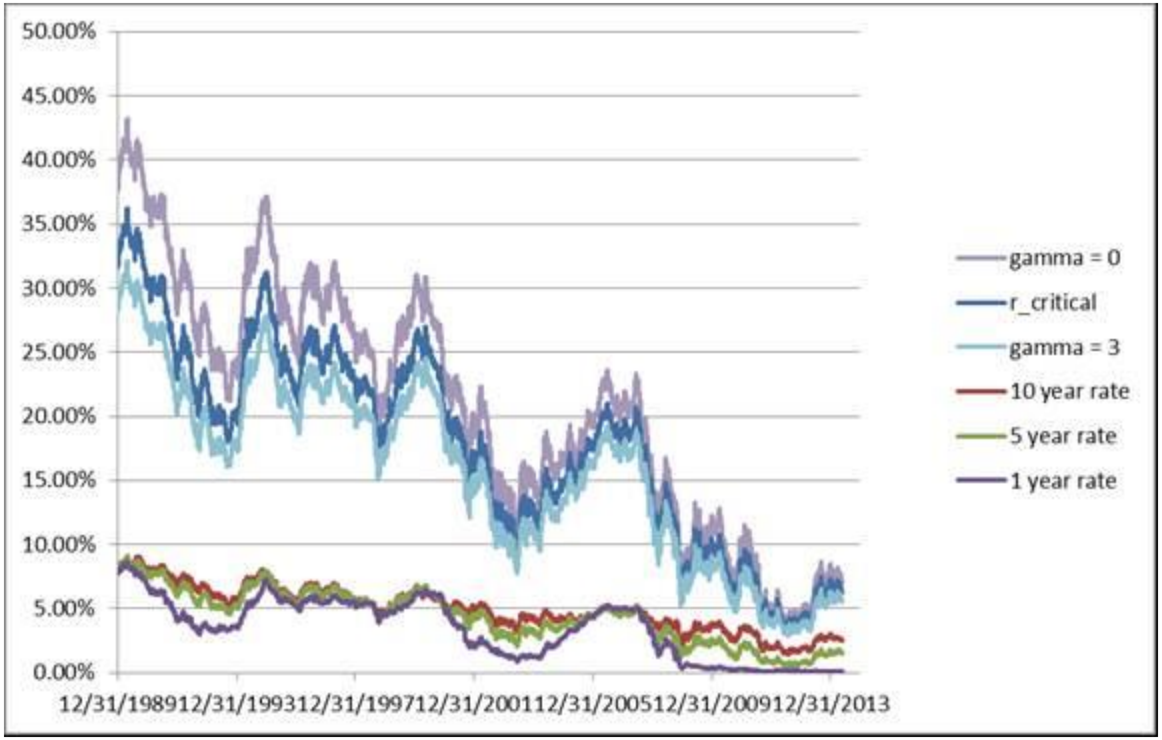
r_{crit} with “0” floor, 25% restriction



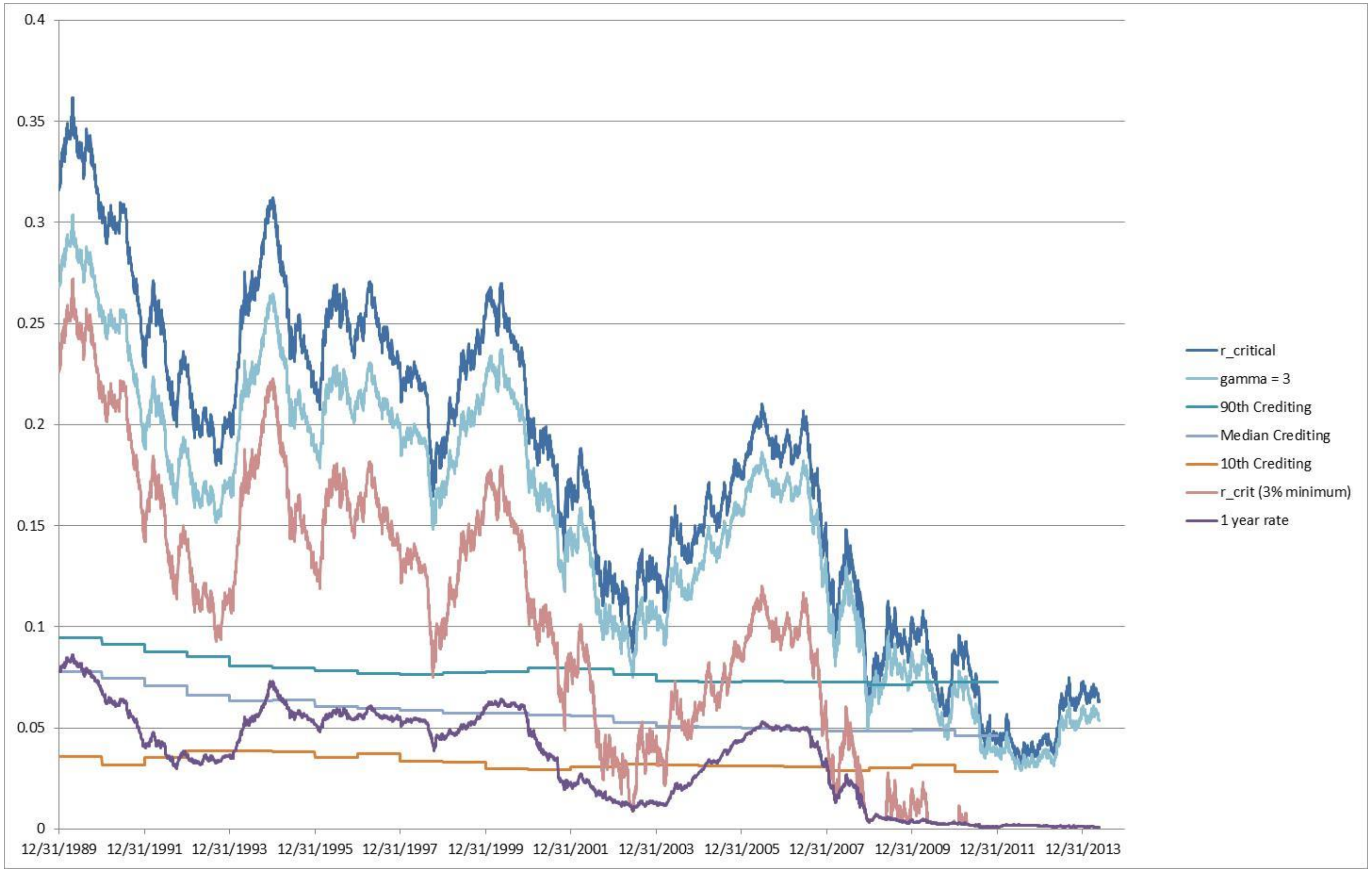
r_{crit} with 3% floor, 25% restriction



r_{crit}^{P} vs. Time for Risk-Averse Policyholders.



Actual and Critical Credited Rates.



Regression Analysis

- Interest Credited vs. Internal and External Rates.

	Coefficients	Standard Error	P-value
Intercept	-$3,547,190	$446,845	2.38E-15
Assets	0.006	0.001	5.57E-17
NII on Line	0.076	0.003	4.7E-101
NII Proportional	0.049	0.009	1.59E-07
Short Term	0.288	0.019	1.38E-49
5 Year	-1.634	0.073	3.9E-108
10 Year	2.208	0.065	2.5E-234

Conclusions

- Optimal Strategy:
  - IC credits r_{crit} then r_{min}
  - PH transfers out of MM if $r_c \geq r_{crit}$ and into MM otherwise.
- Restricted Arbitrage Opportunities are possible.
- Companies tend to credit based on external rates, not company specific NII rates.