A Few Myths in Quantitative Finance

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Outline

I. Data
II. Models
III. Hedging
IV. Behavioral Finance
V. Social Utility
I. Statistics/Historical Data
Sharpe ratio myth:
“High Sharpe ratios are rare”
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- Sharpe ratio $> 1$ is good, $> 2$ is exceptional (?)
- Example of a strategy over 1 year
  - with a Sharpe ratio $> 3$
  - no losing month
Just go long SPX in 1995!
Jump myth:
“Jumps are mostly downwards”
Are big moves really down?

Two moves of more than 10%, both up!
Close up

Two moves of more than 10%, both up!
# Biggest historical returns

Over the last 100 years, top 10 returns, 8 out of 10 up!

<table>
<thead>
<tr>
<th>Dates</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/28/1929</td>
<td>-12.94%</td>
</tr>
<tr>
<td>10/30/1929</td>
<td>12.53%</td>
</tr>
<tr>
<td>06/22/1931</td>
<td>10.51%</td>
</tr>
<tr>
<td>10/06/1931</td>
<td>12.36%</td>
</tr>
<tr>
<td>09/21/1932</td>
<td>11.81%</td>
</tr>
<tr>
<td>03/15/1933</td>
<td>16.61%</td>
</tr>
<tr>
<td>09/05/1939</td>
<td>11.86%</td>
</tr>
<tr>
<td>10/19/1987</td>
<td>-20.47%</td>
</tr>
<tr>
<td>10/13/2008</td>
<td>11.58%</td>
</tr>
<tr>
<td>10/28/2008</td>
<td>10.79%</td>
</tr>
</tbody>
</table>
Other jump myth:
“Jumps are well modeled by Levy processes”
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“Jumps are well modeled by Levy processes”

- Pitfalls of Levy modeling:
  - Back to normal just after a jump
  - No time clustering of jumps
  - Skew vanishes fast
  - Hawkes processes cluster jumps
Dividend myth:
“Dividends yields are quite stable”
Common dividend modelling

- Known amount on the short term
- Proportionality to the stock price on the long term
Coca Cola example
Properties of dividends curves

- Most of the time non decreasing

- Requires path dependent models to account for crisis impact
Correlation myth: “Highly correlated assets are proxies”
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X and Y are 2 stocks of same volatility: \( \sigma \)

Very highly correlated: \( \rho(X, Y) = 0.99 \)

Are they almost perfect substitutes? \textbf{NO}

\[
\sigma^2_{X-Y} = \sigma^2 + \sigma^2 - 2\rho\sigma^2 \\
\sigma_{X-Y} = \sigma \sqrt{2(1-\rho)} \approx 0.14\sigma
\]

The risk of \( X - Y \) is still 14% of the initial risk!
Correlating levels/increments

\[ X_t = S&P_t, \quad Y_t = S&P_{t+\delta t} \]

Levels very correlated
Increments decorrelated

\[ X_t = S&P_t, \quad Y_t = X_t + \alpha t \]

Levels weakly correlated
Increments fully correlated
Correlation/Causation

- Correlation of A and B is a (linear) measure of co-occurrence
- It may miss a real link between A and B
Skewness myth:
“The skew comes from the skewness of returns”
Dissociating Jump & Leverage effects

\[ t_0 \quad \quad \quad t_1 \quad \quad \quad t_2 \]

\[ x = S_{t1} - S_{t0} \quad y = S_{t2} - S_{t1} \]

- **Variance:**
  \[ (x + y)^2 = x^2 + 2xy + y^2 \]
  - Option prices
  - FWDD variance
  - Δ Hedge

- **Skewness:**
  \[ (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \]
  - Option prices
  - Leverage
  - Δ Hedge
  - FWDD skewness
Define a time window to calculate effects from jumps and Leverage. For example, take close prices for 3 months

- **Jump:**
  \[
  \sum_{i} \left( \delta S_{t_i} \right)^3
  \]

- **Leverage:**
  \[
  \sum_{i} \left( S_{t_i} - S_{t_1} \right) \left( \delta S_{t_i} \right)^2
  \]
Skew comes from leverage
Other skewness myth: “Skewness is easy to estimate”
Other skewness myth: “Skewness is easy to estimate”

- Most samples are below the mean
- Empirical mean is most of the time below the expectation
- Binomial and lognormal martingales examples:
Kurtosis myth:
“Returns high kurtosis are due to jumps”
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“Returns high kurtosis are due to jumps”

- Stock returns are leptokurtic (fat tails)
- Are the fat tails due to changes of volatility or to jumps?
S&P 500 Returns 2001-2014

Kurtosis: 11.7
S&P 500 Volatility Normalized Returns

SPX Index Normalized Returns Chart from 2001-01-02 to 2013-12-31

Histogram of normalized daily returns

Kurtosis: 4.8
S&P 500 Returns May 2008 - May 2010

Kurtosis: 7.85
S&P 500 Volatility Normalized Returns

SPX Index Normalized Returns Chart from 2008-04-29 to 2010-04-30

Histogram of normalized daily returns

Kurtosis: 3.41
II. Models
Calibration myth:
“A calibrated model prices well”
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- Bad implied dynamics
- Example: Heston has overblown volvol,
- Due to volvol\*correlation as only way to produce skew
- As a consequence, Feller condition is violated and volatility reaches 0
Heston model:

\[ dv_t = \kappa (\theta - v_t) \, dt + \omega \sqrt{v_t} \, dW_t \]

Calibrated to S&P on July 17th 2014:

\[ \kappa = 2.09 \]
\[ \theta = 0.043 \]
\[ v_0 = 0.01 \]
\[ \omega = 59.8\% \]
SABR myth: “SABR manages smile risk”
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“SABR manages smile risk”

- Backbone: behavior of ATM vol as a function of spot
- Model claims to dissociate fitting to the skew from fitting to the backbone
- Managing Smile Risk: NO
2 fitting models

- **SABR A**
  \[
  \begin{align*}
  dF &= \alpha \cdot dW \\
  d\alpha &= \nu \alpha \cdot dZ
  \end{align*}
  \]

- **SABR B** calibrated to A
  \[
  \begin{align*}
  dF &= \alpha' F \cdot dW \\
  d\alpha' &= \nu' \alpha' \cdot dZ
  \end{align*}
  \]

Same skew

Different backbones

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\nu$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>0.1</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0.1</td>
<td>1</td>
<td>0.175</td>
<td>-0.58</td>
</tr>
</tbody>
</table>
\[ \sigma_{ATM}^{F_{T_1}, T_2} \]

- Scattered plot
- Average backbone

**Same skew \Rightarrow** similar vol dynamics\text{ in average } = \text{ LVM vol dynamics}
Interest rate myth:
“Many factors are needed”
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- Analysis of interest rate data
- PCA of the yield curve
- Mean reversion?
- Need for tools to analyze the data and conditional behavior
- Few dimensions with conditioning preferable to many blind ones

- US rates PCA pre and post crisis
PCA pre crisis
PCA post crisis
Arbitrage Pricing Theory myth
“APT is a multi-factor model”
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“APT is a multi-factor model”

- Assume the factors are tradable

- NP = Numeraire Portfolio, associated to measure “P”

- No risk premium for the noise => NP is in the space spanned by the factors

- The factors’ risk premia locate NP in this space

- Reduces to 1 factor! NP plays the role of the Market Portfolio in the CAPM
\[ X = \Sigma \beta_i F_i + \varepsilon \implies RP_X = \Sigma \beta_i RP_{F_i} \]

\[
NP = \Sigma \alpha_i F_i, \quad \alpha = V^{-1} RP_F \\
RP_X = \frac{CoV(r_X, r_{NP})}{Var[r_{NP}]} \\
RP_{NP} = CoV(r_X, r_{NP}) = \Sigma \beta_i RP_{F_i}
\]
Volatility spike myth
“Volatility jumps up during a crash”

VIX “Jumps” are more like explosive rallies which extend over a few days
III. Hedging
Calibration myth: “Calibrate and price”
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- Calibration without a hedge is pointless
- Examples:
  - droption
  - spread option
  - albatross
  - variance swap adjustment
H2

- Need to measure the “hedgeability” of a claim
Risk management myth:
“Cancel the Greeks to cancel the risk”
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- Greeks culture: cancel a scalar sensitivity
- Depends on what is perturbed
- Match a risk profile (a shape) instead

- Superbucket analysis with Functional Ito Calculus
Asian Option Hedge

Robust volatility hedge with \( PF = \int\int \alpha(K,T) C_{K,T} \, dK \, dT \)

\[
\alpha(K,T) = -\left( \frac{\partial h(K,T)}{\partial t} + \frac{1}{2} \frac{\partial^2 (v_0(K,T) h(K,T))}{\partial x^2} \right)
\]

\( h \) is the conditional expectation of the functional Gamma
IV. Behavioral Finance
Risk neutrality myth:
“Risk neutrality is a psychological attitude wrt risk”
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“Risk neutrality is a psychological attitude wrt risk”

- Risk neutrality: carelessness about uncertainty?

\[ 1 \text{ A} \text{ gives either } 2 \text{ B} \text{ or } 0.5 \text{ B} \Leftrightarrow 1.25 \text{ B} \]

\[ 1 \text{ B} \text{ gives either } 0.5 \text{ A} \text{ or } 2 \text{ A} \Leftrightarrow 1.25 \text{ A} \]

- Cannot be RN wrt 2 numeraires with the same probability

\[ \text{Sun: } 1 \text{ Apple} = 2 \text{ Bananas} \]
\[ 50\% \]
\[ \text{Rain: } 1 \text{ Banana} = 2 \text{ Apples} \]
\[ 50\% \]
Behavioral finance myth
“BF is cute but useless”
Behavioral finance myth
“BF is cute but useless”

Many relevant themes:

- Anchoring
- Framing
- Endowment effect
- Distortion of small probabilities
- Disposition effect
- Overconfidence

For option pricing, regret aversion is central
Regret Aversion

- Real motive for buying derivatives
- Decisions are taken to minimize regret, not to maximize utility
Regret Aversion

- You receive an Apple share as a gift. As you have no view in Apple, you sell it at market value, say $400.
One month later it moves to a) $500 or b) $300. Which case makes you happier?
Regret Aversion

- You receive an Apple share as a gift. As you have no view in Apple, you sell it at market value, say $400. One month later it moves to a) $500 or b) $300. Which case makes you happier?

- Probably b) as it makes you feel smart
- Case a) generates regret
- The desire to capture opportunities may make you overpay for optionality
Initial Position

[Graph showing a linear relationship between price and some variable, marked with a point at $400.]
Hedged Position

![Graph showing a hedged position with a sell point at $400.]
Regret Aversion
Regret Aversion

- Regret aversion creates demand for convexity

  Pushes option prices up

- Explains partly volatility risk premium
Toy Model

- Utility depends not only on wealth but also on regret
- Simple utility function:
  \[ R(X,H) = U(X+H) + V(H) \]
  \[ = -\exp(-.1(X+H) - .8 \exp(-.1H)) \]

X: initial exposure
H: hedge
Hedge when exposure $X(S) = S - S_0$
Impact on the skew
V. Social Utility
High frequency trading myth: “High frequency trading provides liquidity”
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- The “providing liquidity” argument
- Exploit information: fast front running
- Provoke a situation:
  - placing fake orders
  - punching through liquidity holes to force trades from VWAP replicators

- Reward of market makers:
  - Should be for risk taking (inventory risk)
  - Not for private information
Derivatives/Innovation myth: “Derivatives reduce risk”
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- Are derivatives solving the client’s problem or the bank’s problem?
- Derivatives should reduce client’s risk
- Instead they often used to
  - Express a view
  - Avert regret
- They are “80% bought and 20% sold”
- Portuguese railroad example
Euribor 3m 2000-2007
Coupons

Receive: 4.76%

Pay: 1.76% + Spread

Spread = Max[0, Previous Spread + 2*Max(2%-Euribor,0) + 2*Max(Euribor-6%,0) – Digital Coupon]

Digital Coupon = 0.50% if 2% < Euribor < 6%; 0% otherwise
Realized path

Table:

<table>
<thead>
<tr>
<th>Notional</th>
<th>Total Cashflow</th>
<th>Realized Cashflow</th>
<th>Projected Cashflow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-596.54</td>
<td>-69.81</td>
<td>-526.73</td>
</tr>
</tbody>
</table>

Graph:

- Y-axis: Euribor (3M %)
- X-axis: Year (2008 to 2020)
- Data points from 2008 to 2020
- Projected Cashflow from 2008 to 2020

Cashflow chart:

- Y-axis: Cashflow
- X-axis: Year (2008 to 2020)
Conclusion

- Quantitative finance is fraught with misconceptions
- Some lead to disastrous actions
- Derivatives often bought and sold for wrong reasons
- A lot of pricing, not much hedge, very little purpose
- Education and tools badly needed